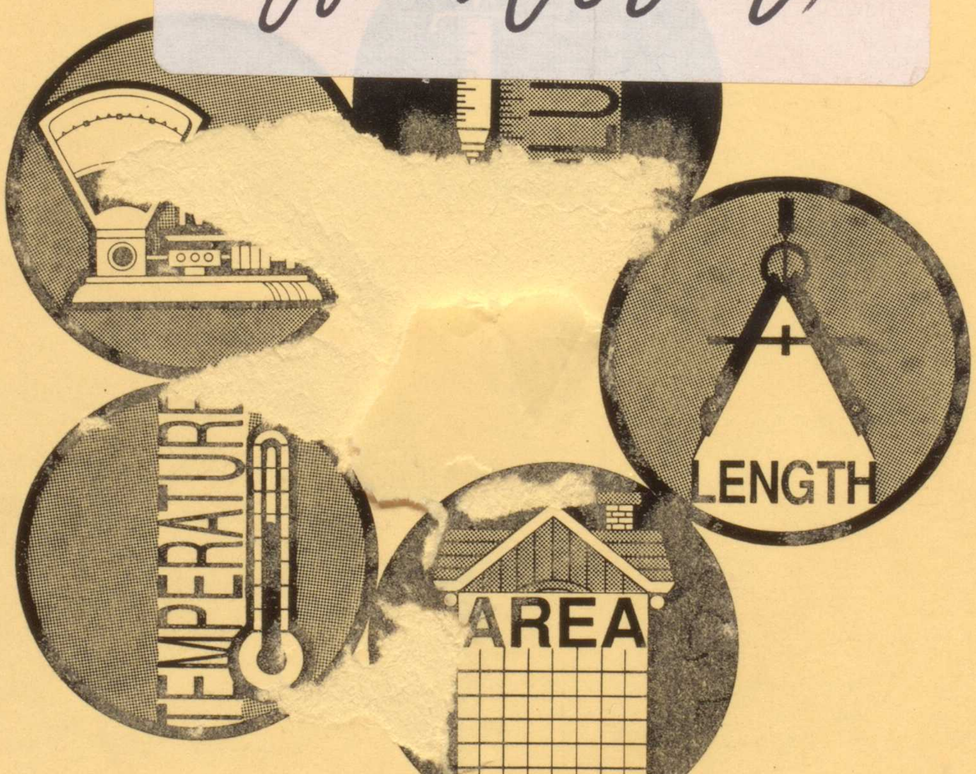
METRICS MANUAL



COLLEGE VCC
King Edward
Campus

VCC - KING EDWARD CAMPUS

MATHEMATICS DEPARTMENT

Prepared by: Wayne Wilson

Research by: K. Oberding

Typing: R.P.M. Lee, Anne Rose

Editing: E. MacLeod

Illustrations: W. Wilson

Revised by: Wayne Wilson, Wayne Ko, 1988

Edited by: J. Cockell, M. Rosati, M. Ko

Cartoons: W. Wilson

Diagrams: W. Ko

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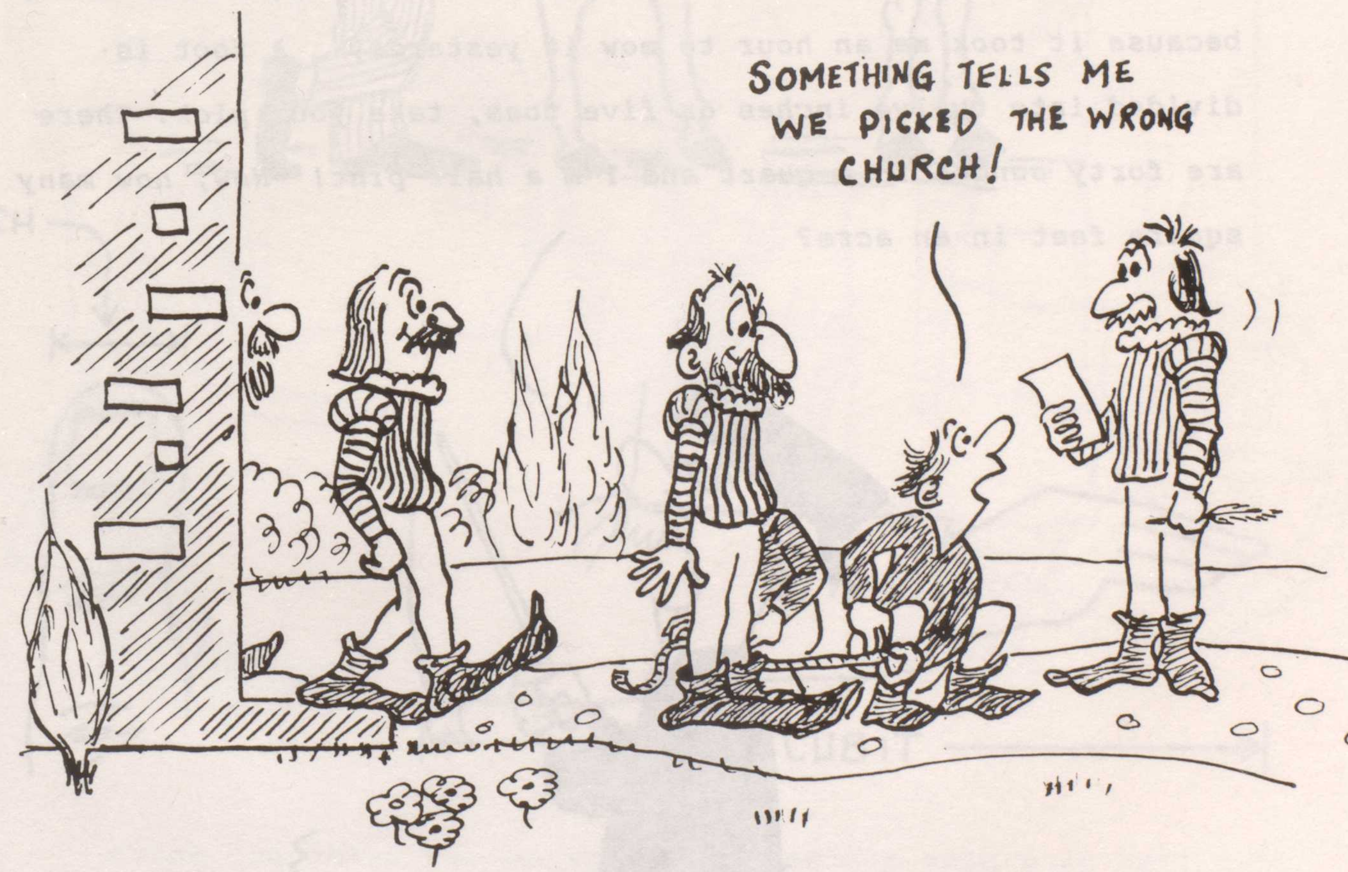
HISTORY OF THE ENGLISH SYSTEM

The English system of measurement evolved much like the English people. It is a polyglot of measures from different systems extending back even beyond the Roman invasion. The Saxons, the Danes, the Normans, each contributed to the confusion after the Romans departed. As a result, many of the English system's units bear no sensible relationship to one another. Furthermore, many of the units lacked standardization until more recent times. Linear measure was originally based on the lengths of the various body parts (which can vary considerably according to the size of a person). Some of the earlier attempts at standardization were quite humorous. A "legal" foot was once declared to be the average of the right foot lengths of the first sixteen men to exit from a particular church on a Sunday morning (the sum of the sixteen foot lengths divided by sixteen).

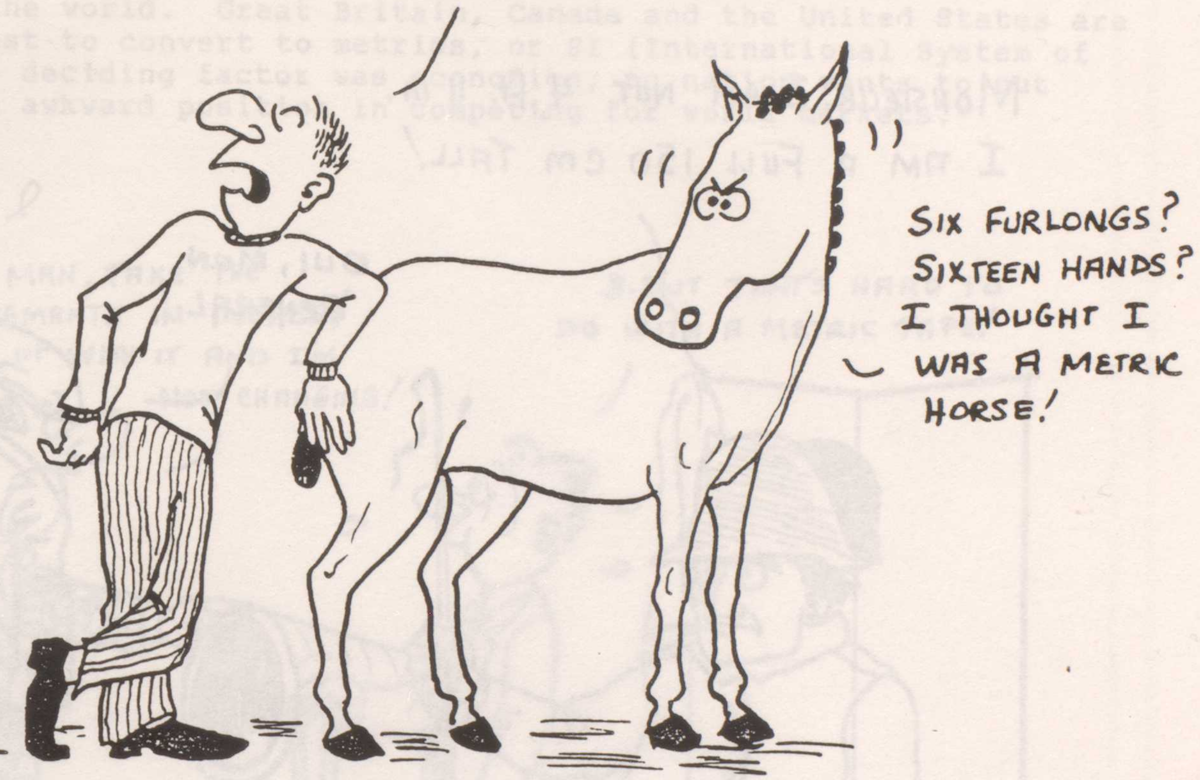


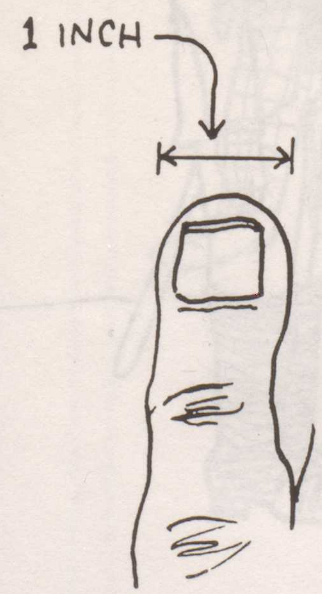
In the evolution of the English measurement system, any new unit deemed useful by a particular ruler, or his advisors, was often added to the list without alteration or replacement of the older units.

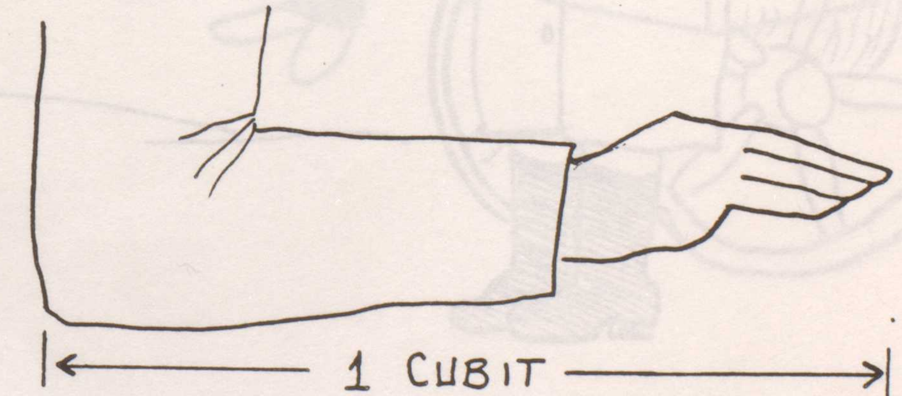
Thus, Henry I of England first introduced the yard as an aid in measuring cloth. The yard was defined as the distance between Henry's nose and the tip of his thumb (with his arm out-stretched sideways). Here are some of the more confusing English measurements:

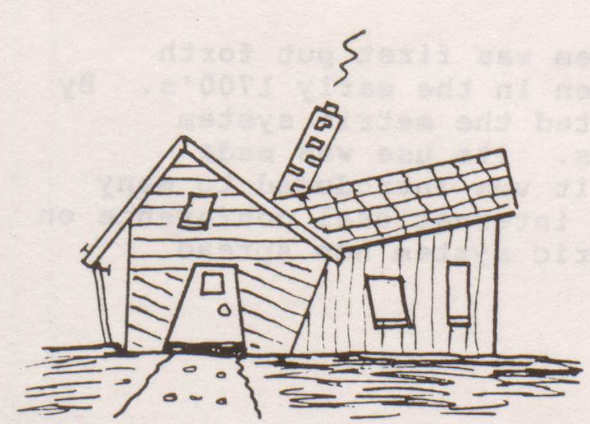
There's the short ton, the long ton, and the wun tun. Sixteen pecks make one bushel.... or is it eight pecks? A gallon is undoubtedly 160 ounces, except in the United States. There are two cups in a pint.... or is it two and a half? There's dry ounces, liquid ounces, and gold ounces. I once caught an ounce in Central Asia. A yard is divided into three parts called feet. But it seems to me that a yard must be bigger than that, because it took me an hour to mow it yesterday. A foot is divided into twelve inches or five toes, take your pick. There are forty ounces in a quart and I'm a half-pint! Now, how many square feet in an acre?

MY HORSE STANDS 16 HANDS HIGH.
HE CAN RUN 6 FURLONGS IN $1\frac{1}{2}$ MINUTES!









FIRST METRIC HOUSE
CONSTRUCTED IN CANADA

HISTORY OF THE METRIC SYSTEM

The idea of a base ten measuring system was first put forth independently by both a Dane and a Frenchmen in the early 1700's. By 1799, the French, under Napoleon, had adopted the metric system developed by the French Academy of Sciences. Its use was made compulsory in France in 1837 and in 1840, it was introduced to many other countries by general agreement at an international conference on weights and measures. Since then, the metric system has spread

throughout the world. Great Britain, Canada and the United States are among the last to convert to metrics, or SI (International System of Units). The deciding factor was economics; no nation wants to put itself in an awkward position in competing for world markets.

UNIT 1

UNIT ONE: SOME BASICS

LINEAR MEASURE

The basic unit for length is the metre (abbreviated as m), which is a little more than a yard. One metre is approximately 39.4 inches. The metre was first standardized in 1791 by the Paris Academy of Science as one ten-millionth part of one quarter of the circumference of the earth along the Prime Meridian. It has been re-standardized several times since then. The latest standardization, by the International Bureau of Weights and Measures in 1960, established the metre as 1 650 763.73 wavelengths of Krypton-86 as measured in a vacuum.

Having established what a metre is, let us now look at the prefixes which are used to denote multiples and fractions of a metre:

kilo	-	1000 of
hecto	-	100 of
deka	-	10 of
deci	-	0.1 of
centi	-	0.01 of
milli	-	0.001 of

These are the most commonly used prefixes in the mass and capacity tables, as well as in length. For the language scholars, the prefixes for the multiples of metres (kilo, hecto, deka) are Greek prefixes, and the prefixes for fractional parts of a metre (deci, centi, milli) are Latin prefixes. Thus, one kilometre is 1000 metres, one centimetre is 0.01 of a metre, and so on.

Next, we should examine the standard abbreviations for measurements of length:

km	=	kilometre
hm	=	hectometre
dam	=	dekametre
m	=	metre
dm	=	decimetre
cm	=	centimetre
mm	=	millimetre

Note that no capital letters were used, nor any periods.

Before we attempt to change from one unit to another, we should make an effort to familiarize ourselves with the most commonly used units. We have already talked about the metre using Imperial measures as a comparison. However, we should not always relate back to the Imperial system if we want to be proficient with the Metric system. Hence, we can think of a metre as roughly half the height of the average door. Using this comparison, we can see that the average person is around 1.75 metres tall. The kilometre is a large measure and is, therefore, a little harder to visualize. A kilometre is about the length of ten football fields set end to end. A centimetre is about the width of the fingernail of your little finger; a pen is roughly 15 centimetres in length. A millimetre is about the thickness of a dime (ten cent coin) or the width of the head of a pin. The other units (hectometre, dekametre, and decimetre) are not commonly used, but we will need them later to help us convert from one unit to another.

As previously mentioned, the most commonly used measures are the kilometre, the metre, the centimetre, and the millimetre. Let us now see when to use which unit. The kilometre is used for great distances, such as the distance between two cities or the length of rivers. For example, the distance from Vancouver to St. John's is 7775 km, the distance from Vancouver to Hope is 145 km, the length of the Fraser River is 1368 km. The metre is used for a wide range of intermediate measures varying from the length of cloth to the height of mountains. For example, floor length drapes are 2.29 m, the height of Mount Everest is 8846.6 m. The centimetre is commonly used to describe smaller measures such as the height of people and the dimensions of paper. For example, the average Caucasian's height is 178 cm and the dimensions of a piece of tissue paper are 21 cm by 22 cm. The millimetre is normally reserved for very small measures such as the diameter of a piece of wire or the thickness of a sheet of plastic. However, the millimetre is also used for construction measurements such as plywood dimensions and room dimensions. For normal everyday use, there is no rigid rule as to when to use which unit, but we should always attempt to pick one that is sensible.

EXERCISE 1A: LINEAR MEASURE

1) What is the meaning of each of the following prefixes?

a) kilo __________

b) milli __________

c) hecto __________

d) centi __________

e) deka __________

f) deci __________

2) What are the abbreviations for the following units?

a) millimetre mm

b) dekametre dm

c) metre m

d) kilometre km

e) hectometre hm

f) decimetre __________

g) centimetre __________

3) Find the error(s) in each of the following and rewrite correctly:

a) 28 KM __________

b) 0.6 m. __________

c) 45 750 cm. __________

d) 8.600 Hm __________

e) 600 k __________

4) What do you think would be the most appropriate unit of SI length for each of the following measures (use the common units only)?

a) the thickness of a vinyl drop sheet ________

b) the distance between Hope and Spuzzum ________

c) the pole vault record ________

d) the length of a boat ________

e) the dimensions of a photograph ________

f) a person's height ________

g) the diameter of a strand of spider web ________

5) Fill in the unit of measure that best describes the given situation (use the common units only):

a) The circumference of Billy's waist is 80 ______.

b) In one hour, Dick can travel 40 ______ by car.

c) The length of a toothpick is about 55 ______.

d) Philbert fell off a 15 ______ cliff and killed himself.

e) Slick lives 5.6 ______ from KEC.

f) Cheryl's tropical plant is 0.93 ______ high.

g) Minnie caught a fish that had a length of 46.3 ______.

CAPACITY

Capacity is the amount of space something takes up. The basic unit of capacity in the metric system is the litre. The litre is the space occupied by a cube that has 10 centimetre sides. Common uses of the litre is to measure the amount of space taken up by liquids. Most of us are familiar with the litre already; we would go to the supermarket and buy one litre of milk. The capacity of aquariums and the amount of gasoline we pump into our car are other examples of when the litre is used. Capacity can also be measured in terms of cubic centimetres, but this will be analyzed in detail in a later unit. For the time being, we will just concentrate on the litre.

As for linear measure, we can apply the prefixes kilo, hecto, etc. to obtain multiples or fractions of a litre. Below is a table of these units, their abbreviations and their relationship to the litre:

kilolitre (kl)	= 1000 litres
hectolitre (hl)	= 100 litres
dekalitre (dal)	= 10 litres
litre (L)	= 1 litre
decilitre (dl)	= 0.1 of a litre
centilitre (cl)	= 0.01 of a litre
millilitre (ml)	= 0.001 of a litre

Note that the abbreviation for litre is a capital L rather than a lower case letter. This is because a lower case l is easily confused with the number one (1). Although the capital L is generally accepted when abbreviating litre, some people prefer to revert to the small l for the other abbreviations, contending that it looks more natural and causes no confusion. Since the Bureau of Weights and Measures accepts either version, it becomes a matter of personal preference. We will use the lower case version for all our examples. We should also note that some people use a lower case l or a lower case *ℓ* in italics to denote litre. These notations are not proper, but we may come across them some time.

The most commonly used units are the litre and the millilitre; the kilolitre is also used, but it is not as common as the other two. We already discussed the litre in some detail. The millilitre is quite small. A teaspoon will hold about 5 ml of liquid. Drug dosages are often measured in millilitres. The kilolitre is fairly large and is, thus, not used very often. One kilolitre would be about the amount of water needed to fill four bathtubs (all the way to the top).

EXERCISE 1B: CAPACITY

1) What are the abbreviations for the following units?

a) millilitre ____________________

b) litre ____________________

c) dekalitre ____________________

d) decilitre ____________________

e) hectolitre ____________________

f) kilolitre ____________________

g) centilitre ____________________

2) What do you think will be the most appropriate SI unit for the following (use the common units only)?

a) The amount of liquid in a cup ________

b) The amount of water in an aquarium ________

c) The amount of milk required for a recipe ________

d) The amount of water in a swimming pool ________

3) Fill in the unit of measure that best describes the situation (use the common units only).

a) Bill had to pump in 40.5 ______ of gasoline to fill his car.

b) Jackie bought a 750 ______ bottle of wine.

c) Paul has a 50 ______ aquarium.

d) Cindy put in 1 ______ of vanilla extract into her muffin mix.

e) The Wilsons consumed 5.5 ______ of milk last week.

MASS

In normal everyday use, the terms mass and weight are used interchangeably. However, there is an important difference between these two terms when we are considering them from a scientific perspective. Mass is the measure of the "quantity of matter"; this quantity is the same no matter where the object is. A more precise scientific definition is that mass is the measure of the inertia of a body; inertia is the tendency for an object to maintain its state of rest or uniform motion in a straight line. Weight is a force caused by gravity (or acceleration) acting on an object. Weight changes depending on the location of the object. For example, an astronaut weighs (one-sixth) less on the moon than he does on the earth since the gravitational pull of the moon is less than that of the earth; his mass, however, remains the same.

The basic unit of measurement for mass is the gram. The gram is the mass of one millilitre (or cubic centimetre) of water at 4 degrees Celsius (the temperature of its maximum density). Of course, for normal everyday use, we can assume that the temperature would have an insignificant effect and just say that one millitre of water has a mass of one gram. As for the other measures, we can add the prefixes kilo, hecto, etc. to obtain multiples or fractions of a gram. The following table summarizes the relationships and the abbreviations of the various units of mass:

kilogram (kg)	= 1000 grams
hectogram (hg)	= 100 grams
dekagram (dag)	= 10 grams
gram (g)	= 1 gram
decigram (dg)	= 0.1 of a gram
centigram (cg)	= 0.01 of a gram
milligram (mg)	= 0.001 of a gram

As we can see, the meanings of the prefixes are the same and the abbreviations follow a similar pattern as before.

The most commonly used units are the kilogram, the gram and the milligram. The kilogram is used to measure fairly heavy objects such as food packages. One litre of milk would have a mass of about 1 kilogram. The average male would weigh about 60 to 70 kilograms and the average female 50 to 60 kilograms. The gram is a small mass and we mentioned that it is the mass of one millilitre of water. A ten dollar bill or a paper clip would have a mass of about one gram. The milligram is an extremely small mass and is difficult for us to detect physically. It is used mainly in scientific analysis and pharmacy. For example, the amount of vitamins and minerals in a serving of cereal would be measured in milligrams. Another commonly used unit for mass is the (metric) tonne (abbreviated as t). This is a very large measure and is often used to measure the mass of something a company buys or sells. The tonne is equal to 1000 kg. For example, a small car (i.e. a Japanese import) would have a mass of about one tonne.

EXERCISE 1C: MASS

1) What are the abbreviations for the following units?

   a) kilogram __________________________

   b) gram __________________________

   c) hectogram __________________________

   d) decigram __________________________

   e) centigram __________________________

   f) dekagram __________________________

   g) milligram __________________________

   h) tonne __________________________

2) What do you think would be the most appropriate SI unit to measure the following (use the common units only including tonne)?

   a) the mass of an army tank ________

   b) the mass of a cup ________

   c) the mass of toxins in a fish ________

   d) the mass a weightlifter can lift ________

   e) the amount of meat needed to feed 10 people ________

3) Fill in the unit of measure that best describes the given situation (use common units only including tonne):

   a) The pill contains 3.5 ______ of antacid.

   b) The letter has a mass of 6.75 ______.

   c) The football player has a mass of 90 ______.

   d) Joe's grain company exported over one million ______ of grain last year.

   e) The average human brain has a mass of 1500 ______.

   f) Mira bought a 1.25 ______ roast for dinner.

UNIT 2

UNIT 2: CONVERTING UNITS BY SLIDING THE DECIMAL POINT

LINEAR MEASURE

Changing from one unit to another is extremely easy when we are dealing with the Metric system. The most common method is to use the sliding decimal. To convert from one unit to another, we follow the following steps:

1) Arrange the units in order from the largest to the smallest as below:

   km hm dam m dm cm mm

2) Start at the given unit and count how many "places" we must move to arrive at the desired unit (do not count the starting point); also remember the direction that we are moving in.

3) Go to the given number and locate the decimal point. Remember that the decimal point is understood and not shown for whole numbers (the understood decimal point is at the end of the number).

4) Move the decimal point the same number of "places" and in the same direction as in step one. Remember to add zeros for any extra place values that we get.

Example 1: 2.87 km = ________ m

1) Arrange the units from largest to smallest:

   km hm dam m dm cm mm

2) We see that the given unit is km and the desired unit is m. Refering to the list, we see that we must move three places to the right:

   km hm dam m dm cm mm

3) We look at the given number and locate the decimal:

   2.87

4) Move the decimal point three places to the right, remember to add zeros to any place that has missing digits:

   2.87

Therefore, we see that 2.87 km is the same as 2870 m.

Example 2: 75 mm = _______________ cm

1) Refer to our list of units arranged in order from largest to smallest:

km hm dam m dm cm mm

2) We see that we must move one place to the left:

km hm dam m dm cm mm

3) We look at the given number and locate the decimal point. We see that in this particular case, there is no decimal point shown. Therefore, we know that the understood decimal point is at the end of the number:

75^

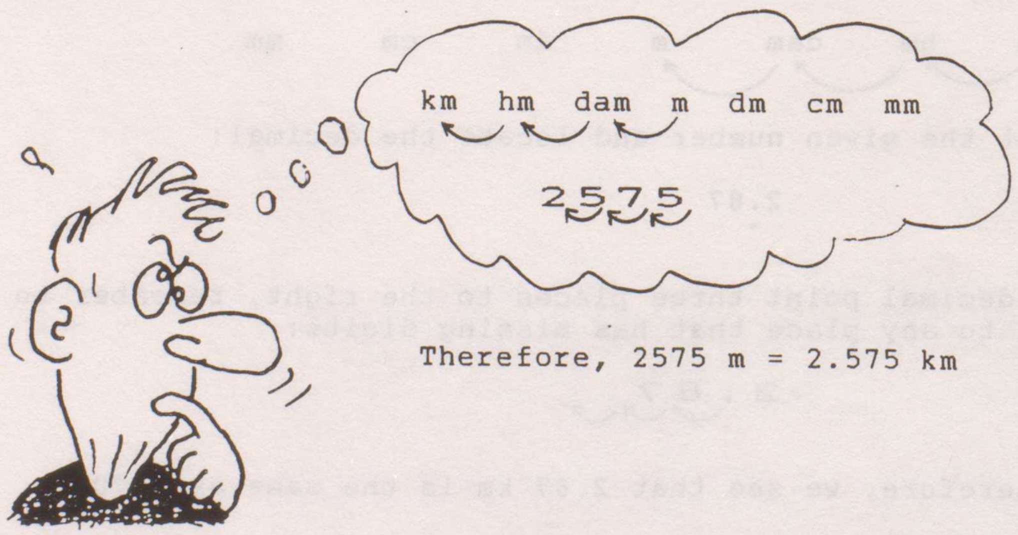
4) Move the decimal point one place to the left; no extra zeros are needed in this case.

7.5

Therefore, 75 mm is the same as 7.5 cm

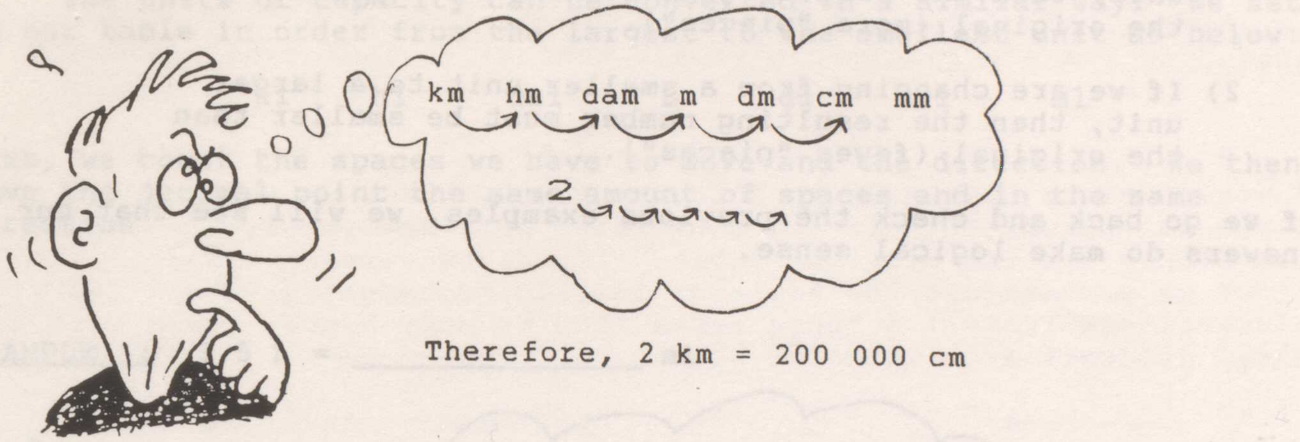
We see that we can actually do everything quite quickly in our head and there is no need to write all the steps down as in the previous examples. In fact, there will come a time when we can simply look at the question and obtain the answer (this will come with practice).

Example 3: 2575 m = _______________ km



Therefore, 2575 m = 2.575 km

Example 4: 2 km = _______________ cm



Therefore, 2 km = 200 000 cm

Note that a space is used to separate the number instead of a comma in the metric system; a comma is too easily confused with a decimal point. In fact, some countries actually use the comma to represent the decimal point. However, we should be aware that some people and texts still use the comma to separate numbers.

Now that we know the method for converting units, we will examine why things work out so nicely. When we refer to our list of units, we notice that adjacent units differ from each other by a factor of 10. For example, 1 km is 10 times greater than 1 hm (recall that 1 km is 1000 m and 1 hm is 100 m), 1 mm is 1/10 of 1 cm (recall that 1 mm is 0.001 m and 1 cm is .01 m),and so on. Hence, we see that when we convert units, each "step" represents a change by a factor of 10, and when we multiply or divide by 10, we can simply move the decimal point one place right if we are multiplying and one place left if we are dividing. For example when we change 2 km to m, we see that km is 1000 times larger than m (3 steps: 10 x 10 x 10), so we multiply by 1000 which is equivalent to moving the decimal 3 places to the right, giving us 2000 m.

One common problem that we run into when we use the sliding decimal point method is that we sometimes get mixed up as to the direction to move. There is an easy way for us to check if we have moved the decimal point correctly. We need to use a little logic to help us predict what happens when we change from one unit to another. When we take a large unit and break it up into smaller units, we see that we end up with more little "pieces". For example if we take a one dollar bill (one item) and change it to cents, we end up with 100 pennies (100 items). When we work in reverse and take small units and build them up into large units, we see that we end up with fewer "pieces". For example we need 100 pennies (100 items) to make a one dollar bill (one item). Using this, we can develop our check as follows:

1) If we are changing from a larger unit to a smaller unit, then the resulting number must be bigger than the original (more "pieces").

2) If we are changing from a smaller unit to a larger unit, then the resulting number must be smaller than the original (fewer "pieces").

If we go back and check the previous examples, we will see that our answers do make logical sense.

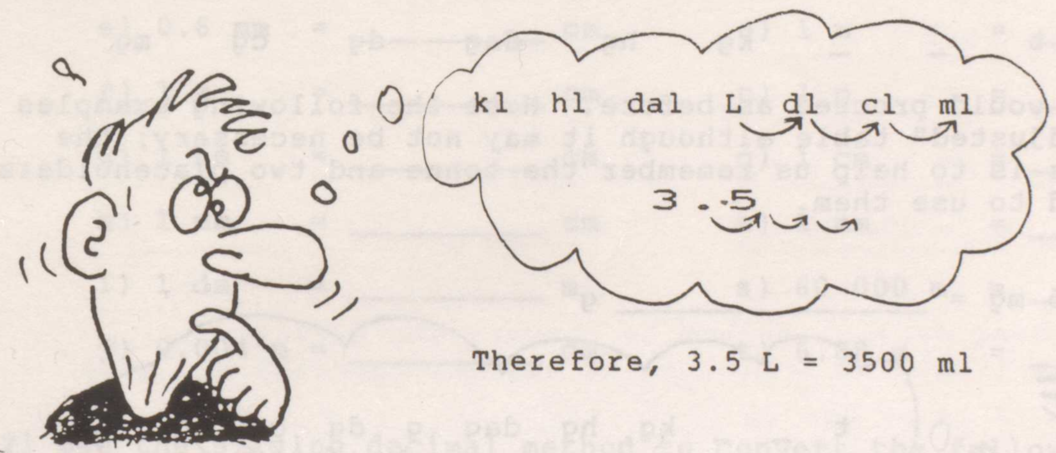
CAPACITY

The units of capacity can be converted in a similar way. We set up our table in order from the largest to the smallest unit as below:

kl hl dal L dl cl ml

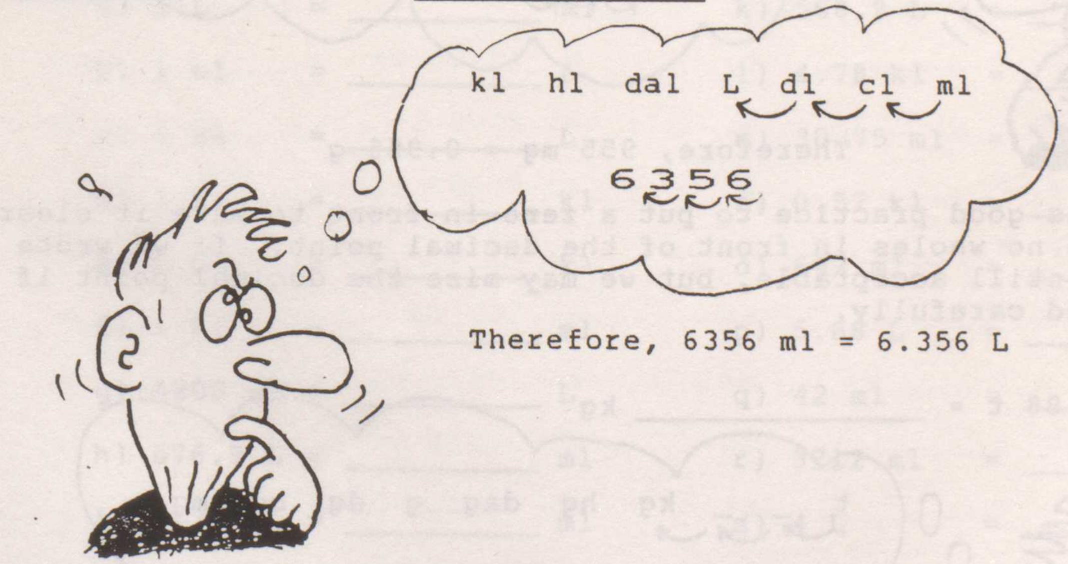
Next, we count the spaces we have to move and the direction. We then move the decimal point the same amount of spaces and in the same direction.

EXAMPLE 1: 3.5 L = ____________ ml



Therefore, 3.5 L = 3500 ml

EXAMPLE 2: 6356 ml = ____________ L



Therefore, 6356 ml = 6.356 L

MASS

Again, the process for converting is very similar. We can arrange the units in order from the largest to the smallest as follows:

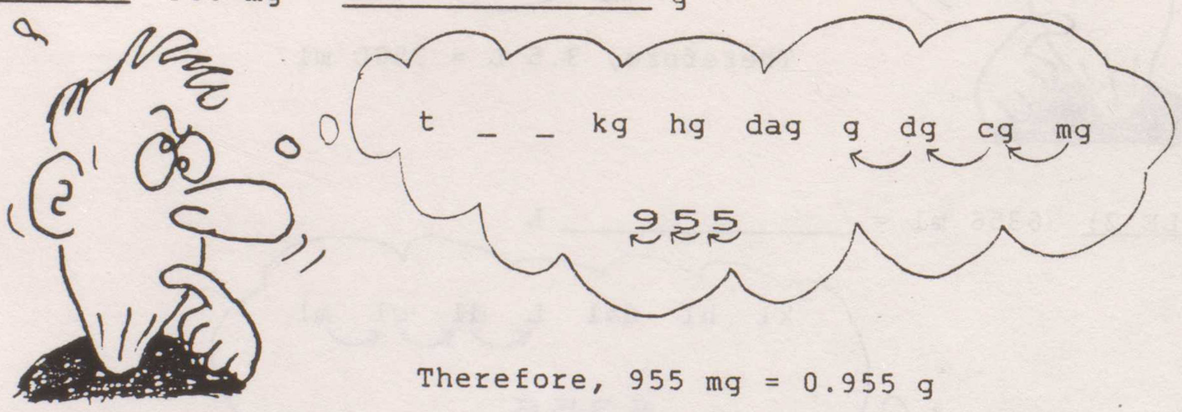
kg hg dag g dg cg mg

However, since the tonne is also frequently used, we must adjust the table to include it. If the conversion does not involve the tonne, we can simply use the previous table. If the conversion involves the tonne we must adjust the table as follows remembering to add two extra placeholders (recall a tonne means 1000 kg and, hence, there must be three "places" from tonne to kg):

t _ _ kg hg dag dg cg mg

To convert, we would proceed as before. Note the following examples all use the "adjusted" table although it may not be necessary; the purpose of this is to help us remember the tonne and two placeholders in case we need to use them.

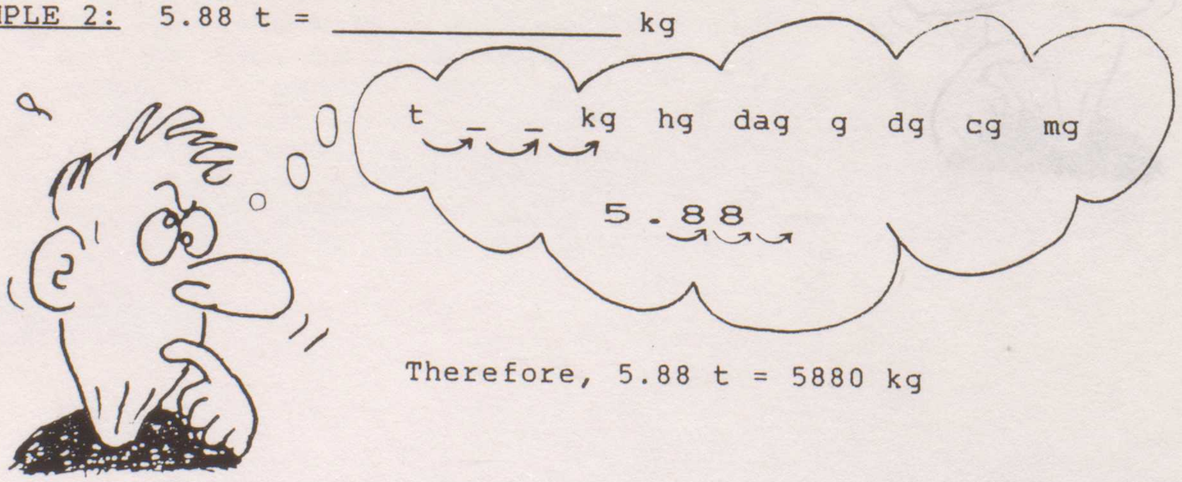
EXAMPLE 1: 955 mg = ______________ g



Therefore, 955 mg = 0.955 g

Note that it is good practice to put a zero in front to make it clear that there are no wholes in front of the decimal point. If we wrote .995 g, it is still acceptable, but we may miss the decimal point if we did not read carefully.

EXAMPLE 2: 5.88 t = ______________ kg



Therefore, 5.88 t = 5880 kg

EXERCISE 2A: CONVERTING COMMONLY USED UNITS

1) Use the sliding decimal method to convert the following linear measures (note that decimetres are in occasional use and have been included in the exercise):

a) 2.3 km = ________ m	k) 676 cm = ________ m
b) 4.78 m = ________ cm	l) 6775 m = ________ km
c) 9.7 cm = ________ m	m) 0.25 km = ________ m
d) 750 cm = ________ m	n) 9487 mm = ________ m
e) 0.6 mm = ________ cm	o) 1 m = ________ mm
f) 1 m = ________ cm	p) 1 m = ________ dm
g) 1 km = ________ cm	q) 1 cm = ________ m
h) 1 mm = ________ cm	r) 1 mm = ________ m
i) 1 dm = ________ m	s) 60 000 m = ________ km
j) 0.004 m = ________ cm	t) 6.88 m = ________ dm

2) Use the sliding decimal method to convert the following capacity measures:

a) 1 L = ________ ml	k) 568.9 L = ________ kl
b) 1 ml = ________ L	l) 4.78 kl = ________ L
c) 1 kl = ________ L	m) 30.75 ml = ________ L
d) 1 L = ________ kl	n) 0.57 kl = ________ ml
e) 1 ml = ________ kl	o) 642 ml = ________ L
f) 1 kl = ________ ml	p) 5.68 L = ________ ml
g) 4200 ml = ________ L	q) 42 ml = ________ L
h) 576.9 L = ________ ml	r) 3212 ml = ________ L
i) 0.32 L = ________ ml	s) 4 L = ________ ml
j) 0.87 ml = ________ L	t) 0.75 L = ________ kl

3) Use the sliding decimal method to convert the following mass measures:

a) 1 t = ________ kg		k) 200 g = ________ kg	
b) 1 kg = ________ g		l) 45 000 g = ________ t	
c) 1 g = ________ mg		m) 103.8 kg = ________ g	
d) 1 mg = ________ g		n) 9.2 t = ________ kg	
e) 1 g = ________ kg		o) 4569 mg = ________ kg	
f) 1 kg = ________ t		p) 5000 kg = ________ t	
g) 7.45 kg = ________ t		q) 400 t = ________ kg	
h) 0.089 g = ________ mg		r) 650.5 kg = ________ t	
i) 0.04 kg = ________ mg		s) 10 g = ________ mg	
j) 85 kg = ________ g		t) 84 g = ________ kg	

4) Use the sliding decimal method to convert the following measurements (there is a mixture of linear, capacity and mass measures):

a) 5.6 t = ________ kg	k) 0.007 cm = ________ mm
b) 0.75 km = ________ m	l) 0.35 mg = ________ g
c) 4.56 L = ________ ml	m) 19.56 cm = ________ km
d) 0.75 kg = ________ g	n) 8000 L = ________ ml
e) 5025 ml = ________ L	o) 0.002 mm = ________ m
f) 2.5 kl = ________ L	p) 35 g = ________ mg
g) 1000 cm = ________ mm	q) 100 mm = ________ m
h) 29 L = ________ ml	r) 345.4 kg = ________ t
i) 0.35 g = ________ mg	s) 22555 m = ________ cm
j) 13 L = ________ kl	t) 13 000 mg = ________ g

EXERCISE 2B: CONVERTING UNITS

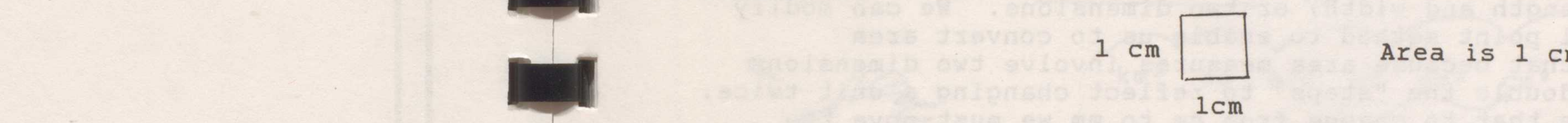
1) Convert the following units using the sliding decimal method. Note that the units in the following exercise include units that are not commonly used.

a) 7.8 dam = ________ m	n) 2.3 kg = ________ g
b) 5.66 hm = ________ cm	o) 6278 ml = ________ dl
c) 0.2 kg = ________ dag	p) 677.6 L = ________ hl
d) 98 dm = ________ mm	q) 45.95 cg = ________ hg
e) 845.1 L = ________ ml	r) 0.233 dam = ________ cm
f) 4500 dg = ________ dag	s) 35.78 dl = ________ kl
g) 63.75 m = ________ dm	t) 676.35 mg = ________ dg
h) 3 km = ________ hm	u) 80 000 ml = ________ dal
i) 99.99 g = ________ cg	v) 0.9 hm = ________ mm
j) 456 kg = ________ dg	w) 6755 kl = ________ cl
k) 34 dal = ________ hl	x) 564.75 hg = ________ dag
l) 0.004 L = ________ cl	y) 35.987 hm = ________ km
m) 0.003 m = ________ dm	z) 4.2 dl = ________ ml

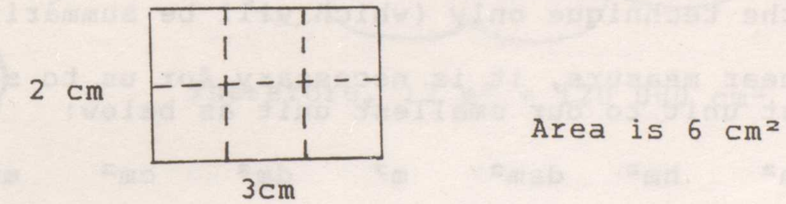
UNIT 3

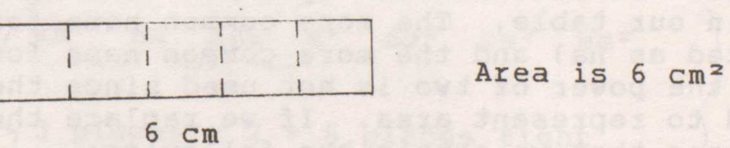
UNIT 3: AREA MEASURE

We can think of area as how much "flat surface" something covers. The basic shape that we use to determine the area of something is the square. For example, a square that has sides that are one cm on each side will have an area of one square centimetre (abbreviated as cm^2). We say "square" centimetres because the basic shape for area is a square. The power of 2 in the abbreviation indicates two dimensions.



If we consider a rectangle with a length of 3 cm and width of 2 cm, we can see that the area of the rectangle is 6 square centimetres (or 6 cm^2). A rectangle that has a length of 6 cm and a width of 1 cm will also have an area of 6 cm^2.





We see that we can find the area of a rectangular shape by multiplying the length with the width ($A = l \times w$). If we consider the above shapes for example, we see:

The square with 1 cm sides: Area = 1 cm x 1 cm = 1 cm^2

The 3 cm by 2 cm rectangle: Area = 3 cm x 2 cm = 6 cm^2

The 6 cm by 1 cm rectangle: Area = 6 cm x 1 cm = 6 cm^2

We will discuss finding the area of something in greater detail in a later unit. Now that we have a notion of what area is, let us consider how to go about changing from one unit to another.

Suppose we wanted to find the relationship between cm^2 and mm^2, we can go about this by considering the area of our square with 1 cm sides. We know that the area for the square can be obtained by our formula $A = l \times w$: Area = 1 cm x 1 cm = 1 cm^2. However, we know that 1 cm is the same as 10 mm and we can therefore, rename the sides to 10 mm. When we find the area using our formula, we see that the area is 100 mm^2 (Area = 10 mm x 10 mm = 100 mm^2).

1 cm [square] Area = 1 cm^2 10 mm [square] Area = 100 mm^2

1 cm   10 mm

Thus, we see that 1 cm^2 = 100 mm^2. We see that area deals with two linear measures (length and width) or two dimensions. We can modify our sliding decimal point method to enable us to convert area measures. We see that because area measures involve two dimensions we merely have to double the "steps" to reflect changing a unit twice. For example, we see that to change from cm to mm we must move the decimal point one place to the right. There are two cm to change and, therefore, one place to the right (for the first cm) and one more place to the right (for the second cm) is the same as sliding the decimal point two places to the right. It is not necessary for us to remember this technical analysis when we convert area measures. We must remember the technique only (which will be summarized later).

As for linear measure, it is necessary for us to set up a table from our largest unit to our smallest unit as below:

km^2 hm^2 dam^2 m^2 dm^2 cm^2 mm^2

For area, however, matters are a little more complex. The two units hm^2 and dam^2 are better known by their other names and should be replaced in our table. The more common name for hm^2 is the hectare (abbreviated as ha) and the more common name for dam^2 is the are. Note that the power of two is not used since these two units are understood to represent area. If we replace these two names in our table, we see that we obtain the following:

km^2 ha   are m^2 dm^2 cm^2 mm^2

The most commonly used units are km^2, ha, m^2, and cm^2. The square kilometre (km^2) is only used for extremely large areas such as the area of a city or country. Large areas such as farm land are usually measured in hectares. Smaller areas such as the area of a floor are measured in square metres (m^2). Extremely small areas such as the area of a piece of paper or coin would be measured in cm^2. The square millimetre is also used, but it is not too common because it is such a small measure.

Let us now summarize the steps for converting area measures using the sliding decimal method and look at a few examples:

1) Arrange the table in order from the largest to the smallest unit as follows (using the more common names for hm^2 and dam^2):

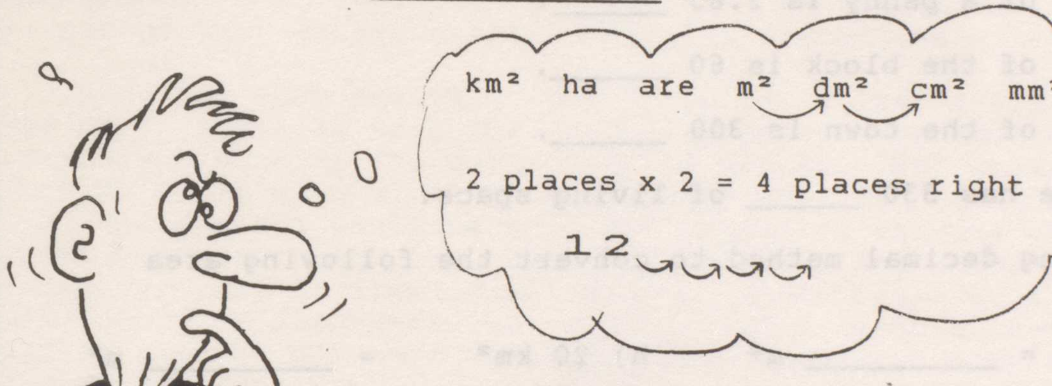
   km^2 ha   are m^2 dm^2 cm^2 mm^2

2) Count the number of "steps" we must move to arrive at the desired unit (do not count the starting point); also remember the direction.

3) Locate the decimal point for the given number.

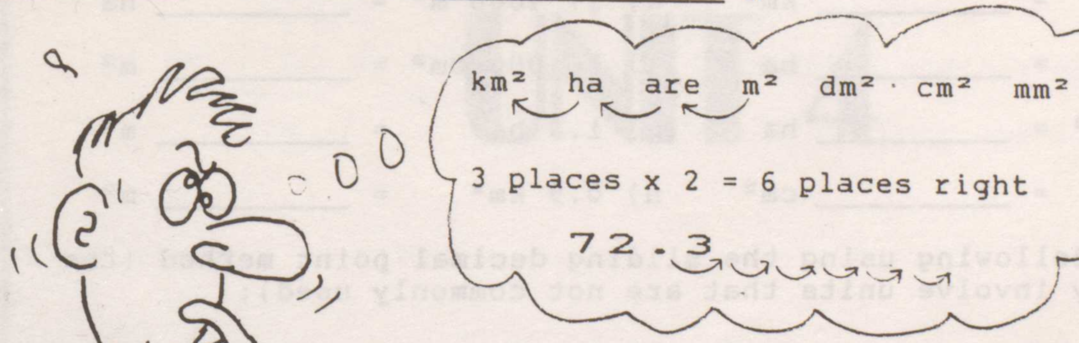
4) Double (multiply by two) the number of steps and move the decimal point in the same direction as step 1.

Example 1: 12 m^2 = ____________ cm^2



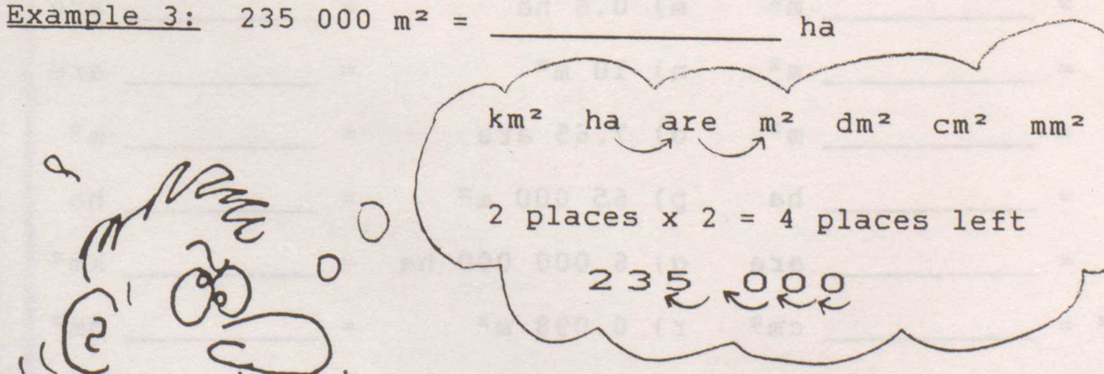
Therefore, 12 m^2 = 120 000 cm^2

EXAMPLE 2: 72.3 km^2 = ____________ m^2



Therefore, 72.3 km^2 = 72 300 000 m^2

Example 3: 235 000 m^2 = ____________ ha



Therefore, 235 000 m^2 = 23.5 ha

EXERCISE 3: AREA

1) Complete with cm^2, m^2, ha, or km^2.

   a) The area of the classroom is about 70 ______.

   b) The area of a penny is 2.83 ______.

   c) The area of the block is 60 ______.

   d) The area of the town is 300 ______.

   e) The house has 330 ______ of living space.

2) Use the sliding decimal method to convert the following area measures:

   a) 2.08 ha = ______ m^2

   b) 0.015 m^2 = ______ cm^2

   c) 40000 m^2 = ______ ha

   d) 750 ha = ______ km^2

   e) 250 m^2 = ______ ha

   f) 0.06 km^2 = ______ ha

   g) 4750 m^2 = ______ cm^2

   h) 20 km^2 = ______ m^2

   i) 560 cm^2 = ______ m^2

   j) 4.27 cm^2 = ______ mm^2

   k) 17 4000 m^2 = ______ ha

   l) 56 000 cm^2 = ______ m^2

   m) 1.8 ha = ______ m^2

   n) 0.9 km^2 = ______ m^2

3) Convert the following using the sliding decimal point method (the questions may involve units that are not commonly used):

   a) 1 m^2 = ______ cm^2

   b) 3800 m^2 = ______ are

   c) 0.6 ha = ______ m^2

   d) 20.1 ha = ______ m^2

   e) 20 km^2 = ______ m^2

   f) 20 km^2 = ______ ha

   g) 2.7 km^2 = ______ are

   h) 2000 mm^2 = ______ cm^2

   i) 0.786 m^2 = ______ mm^2

   j) 4.75 m^2 = ______ mm^2

   k) 1 cm^2 = ______ m^2

   l) 4.7 ha = ______ m^2

   m) 0.6 ha = ______ are

   n) 10 m^2 = ______ are

   o) 7.65 are = ______ m^2

   p) 65 000 m^2 = ______ ha

   q) 6 000 000 ha = ______ km^2

   r) 0.098 m^2 = ______ dm^2

   s) 250 mm^2 = ______ cm^2

   t) 4 750 000 cm^2 = ______ m^2

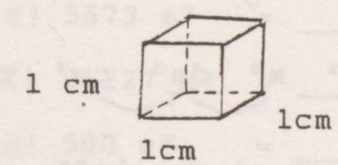
UNIT 4

UNIT 4: VOLUME:

Volume is a measure of how much "space" something occupies. The basic shape for measuring volume is the cube. The volume of a rectangular prism (i.e. a box) can be found by multiplying the length, width and height (i.e. $V = l \times w \times h$). Since volume involves three measures, it is three dimensional. For example, a cube with one centimetre sides will have a volume of one cubic centimetre (abbreviated as cm^3). We say "cubic" centimetre since the basic shape for volume is a cube; the exponent of 3 in the abbreviated form helps remind us that we are dealing in three dimensions.

The two most commonly used units for volume are m^3 and cm^3. The cubic metre is used for fairly large volumes such as the volume of storage bins or the cargo hold of trucks. For example, the average freezer would have a volume of about 1 m^3. The cubic centimetre is much smaller and would be used for measuring volumes of things like small containers and the volume of bricks. For example, a die (singular form of dice) would have a volume close to 1 cm^3. The cubic millimetre is also used, but this is not a frequent event since, we rarely need to use such small measures except in some scientific analysis situations.

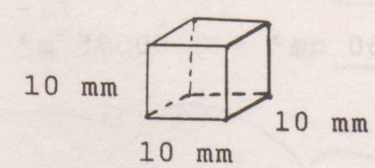
As in the previous unit, we can establish a relationship between cm^3 and mm^3:



Volume = length x width x height

Volume = 1 cm x 1cm x 1cm = 1 cm^3

If we rename 1 cm with 10 mm:



Volume = length x width x height

Volume = 10 mm x 10 mm x 10 mm = 1000 mm^3

We see that since volume is three dimensional, we must triple (multiply by 3) the number of places in moving from one unit to the other. We can summarize the steps for changing volume measure as below:

1) Arrange the table from the largest unit to the smallest unit:

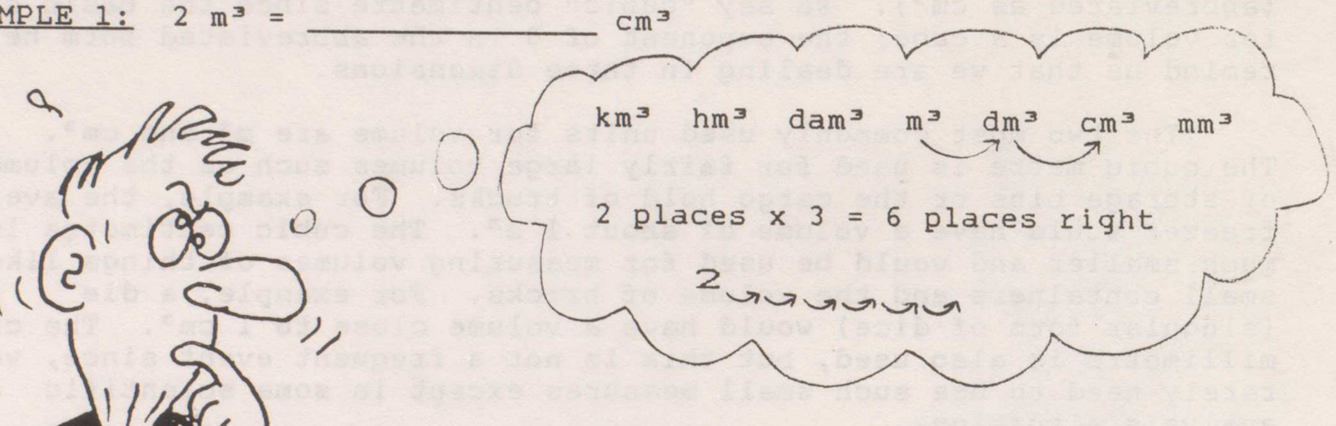
   km^3 hm^3 dam^3 m^3 dm^3 cm^3 mm^3

2) Count the number of places we must move from the given unit to the desired unit and remember the direction.

3) Locate the decimal in our given number.

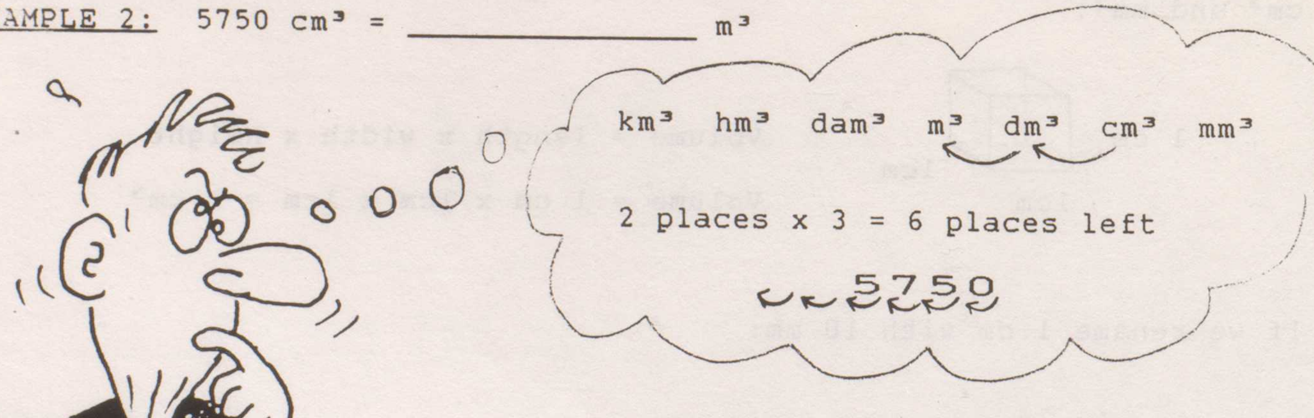
4) Triple (multiply by 3) the number of places we moved in step 1 and slide the decimal point of the given number the same number of places.

EXAMPLE 1: $2\ m^3$ = ______ cm^3



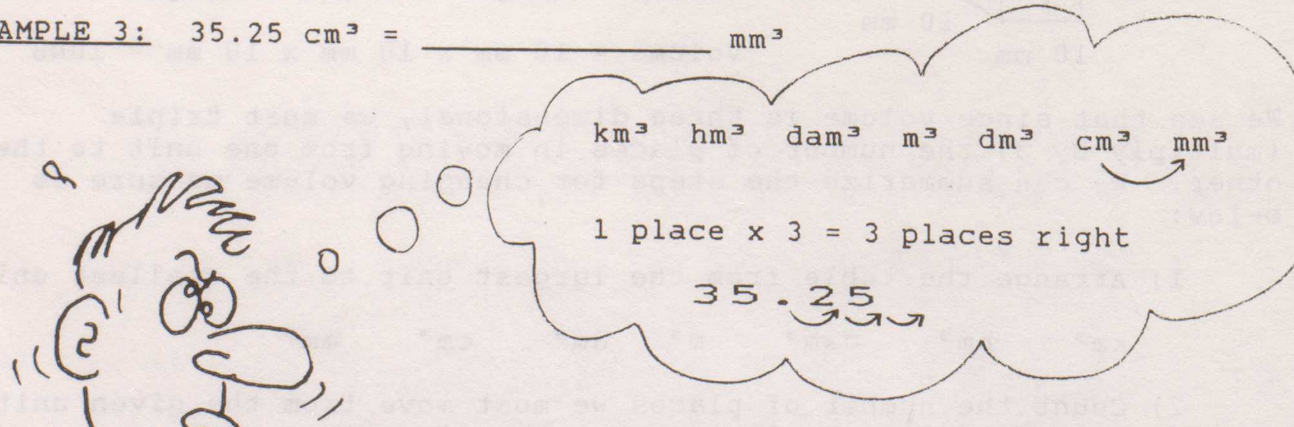
Therefore, $2\ m^3 = 2\ 000\ 000\ cm^3$

EXAMPLE 2: $5750\ cm^3$ = ______________ m^3



Therefore, $5750\ cm^3 = 0.00575\ m^3$

EXAMPLE 3: $35.25\ cm^3$ = ______________ mm^3



Therefore, $35.25\ cm^3 = 35\ 250\ mm^3$

EXERCISE 4: VOLUME

1) Complete with m^3 or cm^3:

   a) The volume of the classroom is about 160 ______.

   b) The bottle has a volume of 750 ______.

   c) The volume of the talk show host's mouth is about 300 ______.

   d) The ship can hold about 4000 ______ of grain.

2) Convert the following volume measures using the sliding decimal method (note: some seldom used units are included to make the exercise more challenging):

   a) $5.7\ m^3$ = __________ cm^3

   b) $345\ cm^3$ = __________ m^3

   c) $4.5\ dm^3$ = __________ cm^3

   d) $0.27\ m^3$ = __________ dm^3

   e) $5673\ m^3$ = __________ hm^3

   f) $0.227\ m^3$ = __________ mm^3

   g) $500\ m^3$ = __________ dam^3

   h) $1\ dam^3$ = __________ m^3

   i) $920\ cm^3$ = __________ m^3

   j) $0.7\ hm^3$ = __________ dam^3

   k) $6.4\ m^3$ = __________ cm^3

   l) $20\ mm^3$ = __________ cm^3

   m) $27.6\ cm^3$ = __________ mm^3

   n) $5\ 000\ mm^3$ = __________ cm^3

   o) $0.575\ m^3$ = __________ cm^3

   p) $0.5675\ cm^3$ = __________ mm^3

   q) $4.5\ hm^3$ = __________ m^3

   r) $52\ 768\ cm^3$ = __________ m^3

   s) $0.6789\ cm^3$ = __________ mm^3

   t) $2000\ cm^3$ = __________ m^3

   u) $1\ m^3$ = __________ cm^3

   v) $0.196\ dm^3$ = __________ mm^3

   w) $500\ cm^3$ = __________ mm^3

   x) $5.74\ m^3$ = __________ mm^3

   y) $0.0003\ cm^3$ = __________ mm^3

   z) $5000\ cm^3$ = __________ m^3

UNIT 5

UNIT 5: RELATIONSHIPS

Volume, capacity and mass are directly related to each other in the metric system. These relationships are exact since the relationships are one-to-one by definition. These exact relationships are one of the many special qualities of the metric system that makes it superior to any other system of measure. By definition, at 4 degrees Celsius:

1 cm³ of water = 1 ml of water = 1 gram of water

Note that for normal usage, the temperature condition can be ignored. Using this basic relationship, we can obtain the following two by converting our units:

1 m³ of water = 1 kl of water = 1 t of water

1 dm³ of water = 1 L of water = 1 kg of water

Using these relationships, we can change from volume to capacity to mass quickly whenever we are dealing with water. If we are only interested in a rough approximation, we can also assume these relationships are true for other liquids that are made up mainly of water (such as orange juice, coffee etc.). For example, a 50 L aquarium would hold 50 kg of water. One litre of milk will have a mass of about 1 kg since milk is made up mostly of water. Note that in the above examples, the temperature is assumed to have a negligible effect.

We should realize that when we deal with volume and capacity relationships only, the equalities are true regardless of the material we are dealing with.

1 cm³ of material = 1 ml of material

1 dm³ of material = 1 L of material

1 m³ of material = 1 kl of material

We see that only the mass relationship is confined to the material being water. For example, a volume of 20 cm³ is equivalent to 20 ml, 5 kl is equivalent to 5 m³. Note that sometimes it may be necessary for us to do some converting before we can establish a relationship. For example, if we were asked to find the number of L in 5700 cm³, we would first change 5700 cm³ to 5700 ml and then to 5.7 L.

EXERCISE 5: RELATIONSHIPS

1) Convert the following using the relationships between volume and capacity:

a) 37 cm^3	=	________ ml	g) 45.7 L	=	________ cm^3
b) 455 m^3	=	________ kl	h) 0.75 kl	=	________ cm^3
c) 32578 ml	=	________ cm^3	i) 4567 cm^3	=	________ L
d) 4.76 kl	=	________ m^3	j) 235 ml	=	________ m^3
e) 8.76 m^3	=	________ kl	k) 5.69 m^3	=	________ ml
f) 53.25 L	=	________ m^3	l) 495 m^3	=	________ L

2) Use the volume, capacity and mass relationships to solve the following problems.

a) What is the mass of three kilolitres of water?

b) A freighter has the capacity to carry 200 000 kilolitres. What is the volume of the hold?

c) If the above freighter is carrying water to Dubai, what would the mass of the water be?

d) A fire truck has a capacity of 80 000 litres of water. What is the volume of the holding tank?

e) If the above fire truck used all the water in its tank (assume it was full at the start) to put out a fire, how many kg of water was pumped out?

f) A cup contains about 250 ml of tea. What is the approximate mass of the tea?

g) Assuming that a reptile is made up of mostly water, what is the approximate volume of a ten kilogram reptile?

h) A scientist did more studies on the reptile in the previous question and found that 80% of its mass is water. What is the volume of water in the reptile?

UNIT 6

UNIT 6: THE IMPERIAL SYSTEM

The Imperial system may appear to be more "sensible" than the metric system for some of us. We should remember that this is only true because we (the older folks) were raised on the Imperial system. The good old days of buying a quart of milk and a pound of butter (for a few cents) are gone. However, just out of interest, we will take a nostalgic look at some of the units that were once in vogue. The younger ones may giggle or shake their heads in disbelief at the "old way", but they still need to be aware of the Imperial system since it will take some time before it is completely replaced by the metric system. Below is a table illustrating the relationship between some of the more commonly used units and their abbreviations.

LINEAR MEASURE:

1 mile (mi.)	=	1760 yards (yd.)
1 mile (mi.)	=	5280 feet (ft.)
1 yard (yd.)	=	3 feet (ft.)
1 yard (yd.)	=	36 inches (in.)
1 foot (ft.)	=	12 inches (in.)

CAPACITY MEASURE (FLUIDS):

1 gallon (gal.)	=	4 quarts (qt.)
1 quart (qt.)	=	2 pints (pt.)
1 pint (pt.)	=	2 cups (c.)
1 cup (c.)	=	8 fluid ounzes (fl. oz.)

MASS MEASURES:

1 pound (lb.)	=	16 ounzes (oz.)
1 ton	=	2000 pounds (lb.)

Note that feet is the plural form of foot, but both are abbreviated in exactly the same way.

The above list is far from complete. If we were to analyze the Imperial system in great detail, we would be shocked by the wide range of units and their "senseless" relationships; how on earth was the relationship of 5280 feet to a mile obtained? If we look back in history at some of the now "obsolete" units, we would see how utterly confusing and silly the system is. For example, 2 mouthfuls = 1 jigger, 2 jiggers = 1 jack (jackpot), 2 jacks = 1 jill, 2 jills = 1 cup etc. The nursery rhyme about Jack and Jill was actually a protest to the King of England (Charles I) on his taxation laws on the "jack"; the rhyme contains three units of measure, the jack, jill, and pail. Gamblers still use the phrase "hit the jackpot" which originated from the King's mishandling of the jack (jackpot) tax. Another example is the use of avoirdupois and troy measures for masses. Troy masses (i.e. troy ounze etc.) are used for precious metals, gems, and drugs, while avoirdupois applies to everything else. Why the distinction? Hence, us "old timers" better be careful when we complain that the metric system is complicated; the "old" system is actually much much more complicated.

EXERCISE 6: IMPERIAL MEASURES

1) Convert the following (students who are not familiar with the Imperial System may want to see the unit on using the unit fraction for a systematic way of converting units):

a) 2.75 mi. = ________ yd.	i) 7 gal. = ________ qt.
b) 676 in. = ________ yd.	j) 3 c. = ________ fl. oz.
c) 5.375 ft. = ________ in.	k) 7.5 pt. = ________ qt.
d) 660 yd. = ________ mi.	l) 450.25 lb. = ________ oz.
e) 45 yd. = ________ in.	m) 335 oz. = ________ lb.
f) 1 mi. = ________ in.	n) 2.3 ton = ________ lb.
g) 60 fl. oz. = ________ c.	o) 8843 lb. = ________ ton.
h) 10 qt. = ________ gal.	p) 180 oz. = ________ lb.

2) The following is designed to compare the time factor, simplicity and accuracy of each system of measure. Note how much time is required to finish each column in the space provided at the bottom.

METRIC	IMPERIAL
a) 24 cm = ________ m	a) 24 in. = ________ ft.
b) 28 m = ________ cm	b) 28 ft. = ________ in.
c) 85 cm = ________ m	c) 85 yd. = ________ ft.
d) 12 m = ________ cm	d) 12 ft. = ________ yd.
e) 18 km = ________ m	e) 18 mi. = ________ yd.
f) 1.25 m = ________ cm	f) 1.25 yd. = ________ ft.
g) 21 120 m = ________ km	g) 21 120 yd. = ________ mi.
h) 6 km = ________ m	h) 6 mi. = ________ yd.
i) 33 cm = ________ m	i) 33 in. = ________ ft.
j) 23 760 m = ________ km	j) 23 760 ft. = ________ mi.
k) 920 mm = ________ m	k) 92 in. = ________ ft.
l) 144 cm = ________ m	l) 144 in. = ________ ft.
m) 12 mm = ________ cm	m) 12 yd. = ________ in.
TOTAL TIME = ________	TOTAL TIME = ________

UNIT 7

UNIT 7: USING UNIT FRACTIONS; LINEAR MEASURE

It is hoped that this unit will prove useful for those students who will be taking a science course in the near future. Anyone who must convert from one system of measurement to another, such as a butcher or a retail grocer, would also benefit. We begin with an Imperial-S.I. conversion table for length:

IMPERIAL	METRIC	METRIC	IMPERIAL
1 in.	= 2.540 cm	1 cm	= 0.3937 in.
1 ft.	= 0.3048 m	1 dm	= 0.3281 ft.
1 yd.	= 0.9144 m	1 m	= 1.094 yd.
			= 3.281 ft.
			= 39.37 in.
1 mi.	= 1.609 km	1 km	= 0.621 mi.

We are now going to apply the concept of multiplying by another name for 1 (this is known as the unit fraction or conversion factor) to change from metric to Imperial or vice versa. Recall that multiplying a number by another name for 1 doesn't change the actual measurement, it just changes the form - which is exactly what we want to do. To use this particular method, we follow the steps below:

1) Write down the given measurement and the unit.

2) Set up our unit fraction taking care to see that the given unit cancels out, and that we are left with the desired unit.

3) Multiply the fractions out and cancel out any unit(s) accordingly.

EXAMPLE 1: 3.5 mi. = ________ km?

1) First, we write down the given measure and unit:

3.5 mi.

2) Next, we set up our unit fraction. We see that we are trying to eliminate mi. and, therefore, we must put mi. in the denominator (bottom). We want to obtain km and, therefore, we put km in the numerator (top). Next we consult our table to see how these two units are related; we see that 1 mile is the same as 1.609 km. Our expression is as follows when we put in the unit fraction:

$$3.5\ \text{mi.} \times \frac{1.609\ \text{km}}{1\ \text{mi.}}$$

3) Next, we multiply and cancel out units as follows:

$$3.5\ \cancel{\text{mi.}} \times \frac{1.609\ \text{km}}{1\ \cancel{\text{mi.}}} = 5.632\ \text{km}$$

Thus, 3.5 mi. = 5.632 km

Note that we could have used the unit fraction 1 km/0.621 mi., but then we would have to divide and that is a little more difficult. When a calculator is used, we see that it does not really matter which unit fraction we use.

$$3.5\ \cancel{mi.} \times \frac{1\ km}{0.621\ \cancel{mi.}} = 5.636\ km$$

Remember that the unit fractions that we are using are only approximations and this causes the slight difference in our two answers. As we can see, the difference is very slight and this would be sufficiently accurate for most circumstances. In fact, we would probably round off the answers to 5.6 km to make things simpler. Science students should note that significant figures should be used for rounding off (consult your Science texts). However, to keep matters simple (since, some of us are not scientifically inclined), we will just round answers off at some convenient place.

<u>EXAMPLE 2:</u> 4 m = ______________ in.?

We should be able to work things out in our heads and arrive at the following:

$$4\ \cancel{m} \times \frac{39.37\ in.}{1\ \cancel{m}} = 157\ in.$$

Notice that in the unit fraction, m is on the bottom in order for us to cancel and in. is on top since it is the unit we want to obtain.

<u>EXAMPLE 3:</u> 5.5 cm = ______________ in.?

$$5.5\ \cancel{cm} \times \frac{0.3937\ in.}{1\ \cancel{cm}} = 2.17\ in.$$

Note that in this example, we could have also used the unit fraction 1 in./2.540 cm instead.

<u>EXAMPLE 4:</u> 3.5 in. = ______________ cm?

$$3.5\ \cancel{in.} \times \frac{2.540\ cm}{1\ \cancel{in.}} = 8.89\ cm$$

Note that in this example, we could have also used the unit fraction 1 cm/0.3937 in. instead.

If we wanted to, we can also use unit fractions to change from one metric measurement to another. In this case, all we need to remember is the meaning of the prefix.

<u>EXAMPLE 5:</u> 2.5 km = ______________ m?

$$2.5\ \cancel{km} \times \frac{1000\ m}{1\ \cancel{km}} = 2500\ m$$

We recall that kilo means 1000 of, and, thus, 1 km = 1000 m. If we wanted to, we could also have used 1 m = 0.001 km (working in reverse, one metre is equal to one one thousandths of a kilometre).

In some instances, we may have to use more than one unit fraction to accomplish our calculation. This is necessary when the relationship between the two given units is not automatically known. The procedure is still the same as before. The only difference is that we must convert to a temporary unit to aid us in our conversion.

<u>EXAMPLE 6:</u> 5345 cm = ______________ km?

We do not know the relationship between cm and km, so we introduce a temporary unit. We can convert cm to m and then from m to km since we know the relationships between these units:

$$5345\ \cancel{cm} \times \frac{1\ \cancel{m}}{100\ \cancel{cm}} \times \frac{1\ km}{1000\ \cancel{m}} = 0.05345\ km$$

Again notice that we could have used slightly different unit fractions.

Science students may find this method extremely helpful for changing between any two units. The following are examples of some typical types of conversions we may encounter. Note that these particular types of questions are not included in the exercises for this unit.

EXAMPLE 7: 45 days = __________ seconds?

We do not know the relationship between days and seconds, so we will take some intermediate steps. We can change days to hours, hours to minutes, and then minutes to seconds:

$$45\ \cancel{days} \cdot \frac{24\ \cancel{h}}{1\ \cancel{day}} \cdot \frac{60\ \cancel{min}}{1\ \cancel{h}} \cdot \frac{60\ s}{1\ \cancel{min}} = 3\ 888\ 000\ s$$

Using this particular method, it is also possible to change items that are made up of several units. We should realize that sometimes the unit we want to cancel may be in the denominator and, therefore, we must set up our unit fraction accordingly.

EXAMPLE 8: 30 km/h = __________ m/s?

First we can change km to m:

$$\frac{30\ km}{h} \times \frac{1000\ m}{1\ km}$$

Next we can change hours to minutes (note that in this case the unit we want to cancel goes on the top of the unit fraction):

$$\frac{30\ km}{h} \times \frac{1000\ m}{1\ km} \times \frac{1\ h}{60\ min}$$

Finally, we can change minutes to seconds (again notice that the unit we want to cancel goes on the top of the unit fraction)

$$\frac{30\ km}{h} \times \frac{1000\ m}{1\ km} \times \frac{1\ h}{60\ min} \times \frac{1\ min}{60\ s}$$

Working things out and cancelling our units:

$$\frac{30\ \cancel{km}}{\cancel{h}} \times \frac{1000\ m}{1\ \cancel{km}} \times \frac{1\ \cancel{h}}{60\ \cancel{min}} \times \frac{1\ \cancel{min}}{60\ s} = 8.3\ m/s$$

Of course, when we work things out, it is only necessary to show our final step; the previous steps should be done in our heads.

EXERCISE 7: UNIT FRACTIONS; LINEAR MEASURE:

1) Convert the following to the indicated unit using the unit fraction method (only one unit fraction is needed):

a) 3.5 in. = __________ cm
b) 2 m = __________ in.
c) 10 yd. = __________ m
d) 7.25 ft. = __________ m
e) 18 mi. = __________ km
f) 3.75 km = __________ mi.
g) 35 m = __________ ft.
h) 5.7 yd. = __________ m
i) 6.3 m = __________ ft.
j) 20 mi. = __________ km

2) Convert the following SI measures using the unit fraction method (more than one unit fraction may be needed):

a) 4.5 mm = __________ m
b) 0.275 m = __________ mm
c) 13 cm = __________ mm
d) 167 km = __________ m
e) 5 km = __________ cm
f) 3.5 km = __________ cm
g) 405 cm = __________ km
h) 4567 mm = __________ km
i) 56 cm = __________ mm
j) 5.78 cm = __________ m

3) Word problems (use the unit fraction method):

a) The driving distance between Vancouver and Calgary is approximately 800 miles. How many kilometres is this?

b) The world record for the long jump is 29 feet 2.5 inches, set by Bob Beamon in the 1968 Mexico Olympics. How many metres is this?

c) Mount Everest is said to be 29 002 feet high. How many metres is this?

d) Sam Stringbean is 2.23 metres tall. Approximately how tall is he in feet and inches?

e) The average marching stride of the soldiers in the Roman Legions was 152.4 centimetres. How many feet is this?

f) A California road sign reads 55 mph (miles per hour). How many kilometres per hour is this?

UNIT 8

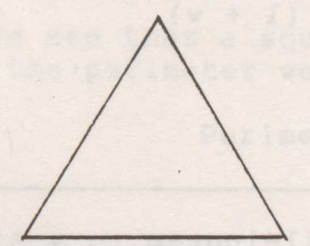
UNIT 8: PERIMETER PROBLEMS

BASIC SHAPES:

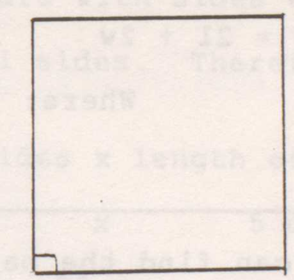
Perimeter is the distance around the edges of a figure (i.e. the sum of the lengths of its sides). To find the perimeter of something, we must find the lengths of all its sides and then add them all together. For some particular shapes, there are formulas we can use to help reduce the work involved. Some of the various formulas are provided below. Note that no formal proofs are given for the formulas; some may be obvious and those that are not will be explained in one of your present or future math classes. Remember to have all measurements in the same units when we find the perimeter of a figure.

Regular Polygons:

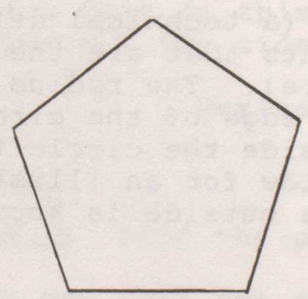
A polygon is a many sided figure. A regular polygon is a figure that has sides that are all the same length and angles that are all the same size. Below are some examples of regular polygons:



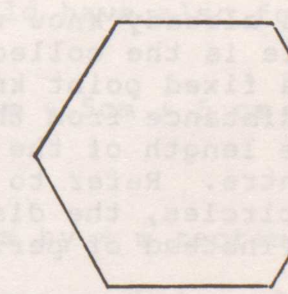
equilateral triangle
(3 sides)



square
(4 sides)



regular pentagon
(5 sides)



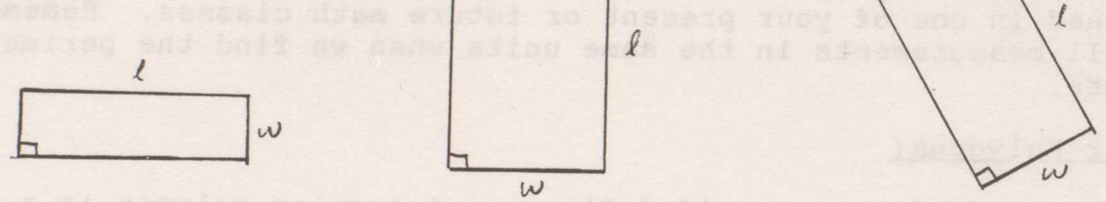
regular hexagon
(6 sides)

For a regular polygon:

Perimeter = number of sides x length of one of the sides

Rectangles:

A rectangle is a "box" shape. All of its corners are "square" (i.e. right or 90° angles). The longer side is called the "length" and the shorter side is called the "width". Note that the square is also labelled as a rectangle; it is the special rectangle that has its length equal to its width. Below are some examples of rectangles:



For rectangles:

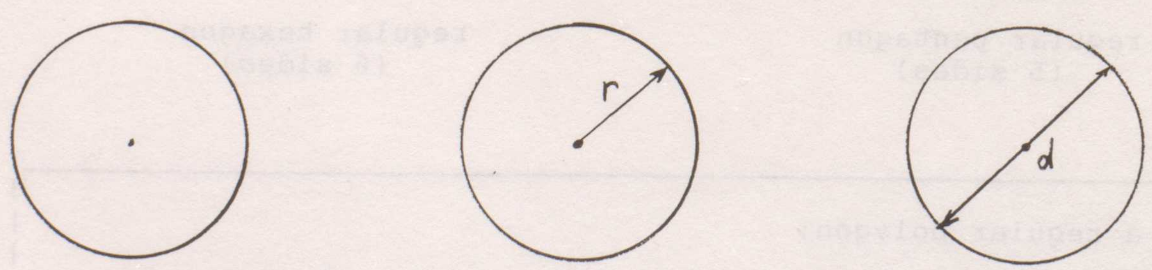
$$P = 2l + 2w \quad \text{or} \quad P = 2\,(l + w)$$

Where: P = perimeter
l = length
w = width

Note that we can find the perimeter of a parallelogram in a similar manner.

Circles:

Most of us already know what a circle is (a technical definition is that a circle is the collection of all points that are the same distance from a fixed point known as the centre). The radius of a circle is the distance from the centre to the edge of the circle; the diameter is the length of the line segment inside the circle that cuts through the centre. Refer to the diagrams below for an illustration of this. For circles, the distance around the outside is known as the circumference (instead of perimeter).



CIRCLE RADIUS DIAMETER

For circles:

$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

We also see that $d = 2r$ or $r = \frac{1}{2}d$

Where C = circumference
r = radius
d = diameter
π = constant (approximately = 3.14)

Note that if we want greater accuracy, we can use more decimal places for the value of π (3.141592654...). However, we will use 3.14 for simplicity.

EXAMPLE 1: Find the perimeter of a square with sides equal to 5 cm.

We see that a square has four equal sides. Therefore, in our case, the perimeter would be:

Perimeter = number of sides x length of side

= 4 x 5 cm

= 20 cm

Thus, a square with 5 cm sides will have a perimeter of 20 cm.

Note that in this example, we could have also found the perimeter by adding all the sides together:

Perimeter = 5 cm + 5 cm + 5cm + 5 cm = 20 cm

EXAMPLE 2: Find the perimeter of a 5 m by 4 m rectangle.

We see that for a rectangle, we have a choice of two forms of the formula. We will use the form P = 2 (l + w).

Perimeter = 2 (length + width)

= 2 (5 m + 4 m)

= 2 (9 m)

= 18 m

Thus, the perimeter of the rectangle is equal to 18 m.

Note that again, we could have found the perimeter by adding the four sides together:

$$\text{Perimeter} = 5\text{ m} + 4\text{ m} + 5\text{ m} + 4\text{ m} = 18\text{ m}$$

EXAMPLE 3: Find the circumference of a circle with a radius of 5 cm.

We notice that we have two formulas for the circumference of a circle. Since the radius is known, we will use the formula with the radius as one of its components: $C = 2\pi r$

$$\text{Circumference} = 2 \times \pi \times \text{radius}$$
$$= 2 \times 3.14 \times 5\text{ cm}$$
$$= 31.4\text{ cm}$$

Thus, the circumference of the circle is (approximately) equal to 31.4 cm. Note that this answer is an approximation since we do not know the exact value of π. If desired, we could round the answer off. If we wanted greater accuracy, we could use a more precise value of π.

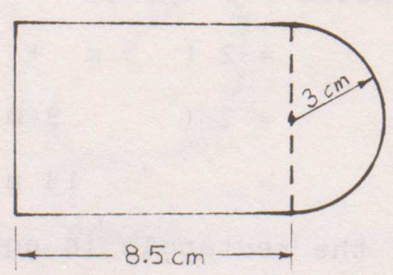
COMPLEX SHAPES:

The previous examples were very simple ones since they are so easy to work with. However, a lot of the time we will get complex shapes that do not have a formula for us to work with. For these cases we can follow the guidelines below to help us solve the problem.

1) Find the lengths of all missing sides. If we have part of a circle, take the desired fractional part of the circumference.

2) Add all the different sides together. We can take short-cuts by multiplying the number of equal sides with the length instead of adding them up separately. Take care not to add sides that are not part of the perimeter.

EXAMPLE 4: Find the perimeter of the following figure:



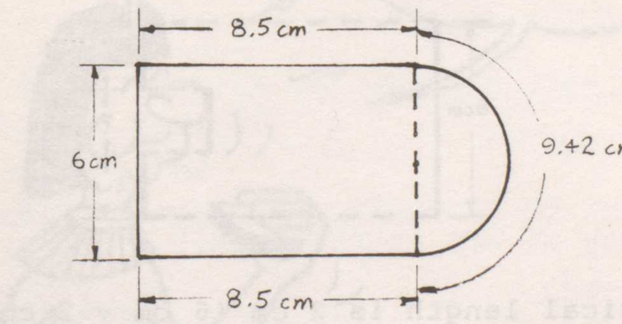
We see that the circular portion of the figure is made up of half the circumference of a circle with a radius of 3 cm. We can find its length as follows:

$$\text{Length of curve} = \tfrac{1}{2} \times \text{Circumference}$$
$$= \tfrac{1}{2} \times 2 \times \pi \times r$$
$$= \tfrac{1}{2} \times 2 \times 3.14 \times 3\text{ cm}$$
$$= 9.42\text{ cm}$$

We see that the left side of the figure must have a length that is double that of the radius of the circle (i.e. its length is the same as the diameter of the circle).

$$\text{Length of left side} = 2 \times 3\text{ cm}$$
$$= 6\text{ cm}$$

We have found all the missing sides and, now, we can label all the sides:



We see that the perimeter of the figure can be found by adding the lengths of all the sides (we will start at the left side and proceed in a clockwise direction):

$$\text{Perimeter} = 6\text{ cm} + 8.5\text{ cm} + 9.42\text{ cm} + 8.5\text{ cm}$$
$$= 32.42\text{ cm}$$

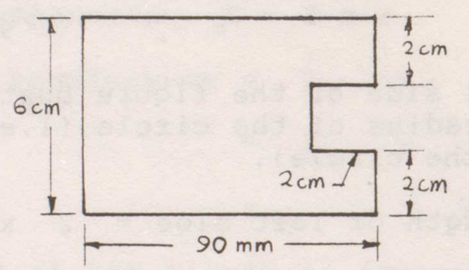
Thus, the perimeter of the figure is equal to (approximately) 32.42 cm.

Note that in the calculation of the perimeter, we could have done it as follows noticing that the top and bottom lengths are the same:

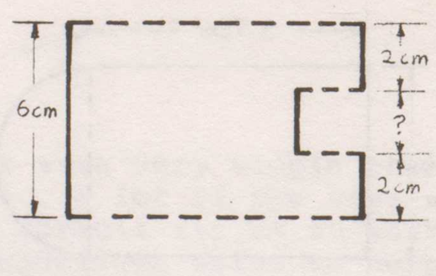
$$\text{Perimeter} = 6\text{ cm} + (2 \times 8.5\text{ cm}) + 9.42\text{ cm}$$
$$= 32.42\text{ cm}$$

We see that either calculation will give us the same answer. Note also that our answer is an approximation in this case since we had to use π, which is not a precise figure. We could have rounded off the answer. Notice as well that the dotted length is not part of the perimeter (since it is inside the figure) and is not included in our calculation.

EXAMPLE 5: Find the perimeter of the following figure:

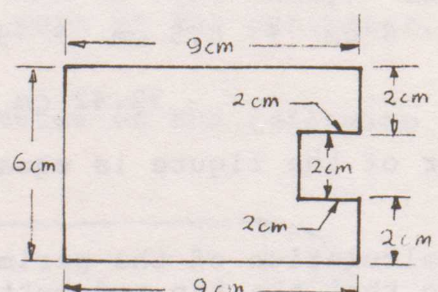


First of all, we must find the lengths of the missing sides:



The missing vertical length is 2 cm (6 cm - 2 cm - 2 cm).

If we put in this length and change all our measures to the same unit (the 90 mm sides is the same as 9 cm), we get:



Therefore, we can find the perimeter by adding all the sides (starting from the left side and going in a clockwise direction:

Perimeter = 6 cm + 9 cm + 2 cm + 2 cm + 2 cm + 2 cm + 2 cm + 9 cm

= 34 cm

We could have simplified this as follows (since we have two 9 cm sides and five 2 cm sides):

Perimeter = 6 cm + (2 x 9 cm) + (5 x 2 cm) = 34 cm

Therefore, the perimeter of the figure is 34 cm.

PTOLEMY DISCOVERING THE VALUE OF PI.

EXERCISE 8: PERIMETER PROBLEMS

Note: for all the questions use 3.14 for π.

1) Find the perimeter or circumference of:

a) a regular hexagon (6 sides) with 5 cm sides.

b) an equilateral triangle with 3.4 m sides.

c) a circle with a diameter of 30 mm.

d) a rectangle with a length of 15 cm and a width of 7 cm.

e) an regular octagon (8 sides) with 6.5 cm sides.

f) a rectangle with a length of 44 cm and a width of 33 cm.

g) a circle with a radius of 5 m.

h) a rectangle with a length of 10 cm and width that is 2 cm shorter than the length.

i) a rectangle with a length of 45.5 mm and a width of 1 cm.

2) Solve the following:

a) The perimeter of a regular heptagon (7 sides) is 63 cm, what is the length of one of its sides?

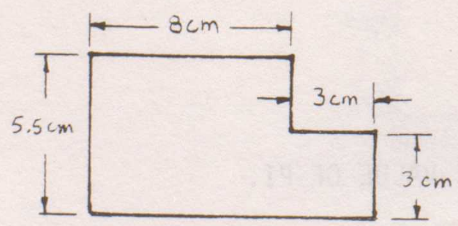
b) Find the radius of a circle whose circumference is 62.8 m.

c) A regular polygon with 2.5 cm sides has a perimeter of 37.5 cm. How many sides does the polygon have?

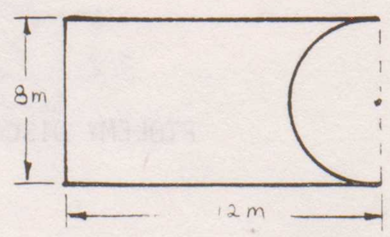
d) The circumference of a circle is 17.898 m. What is the diameter of the circle?

3) Find the perimeter of the following figures:

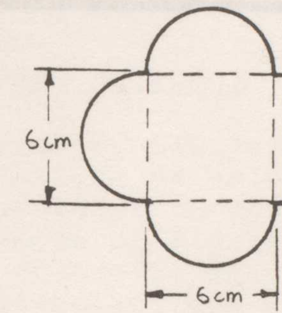
a)



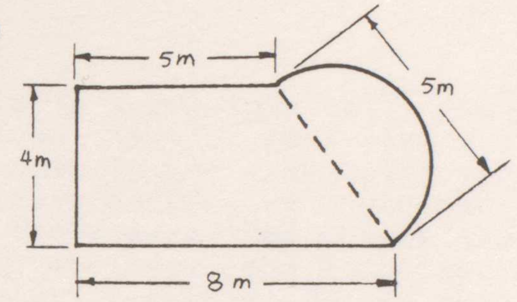
b)



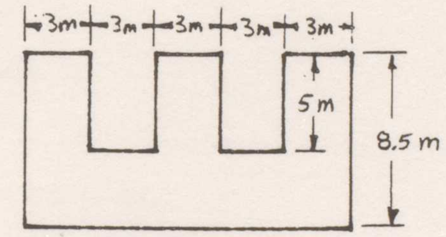
c)



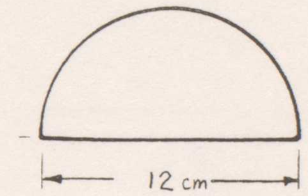
d)



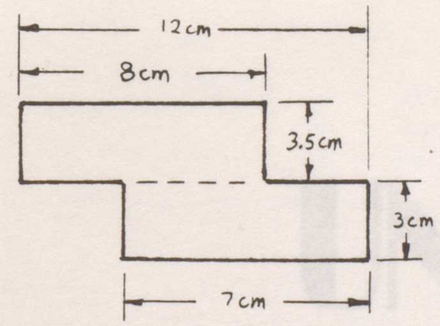
e)



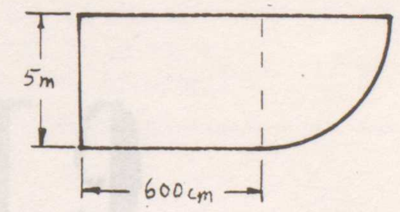
f)



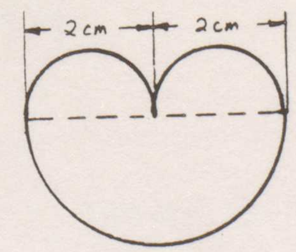
g)



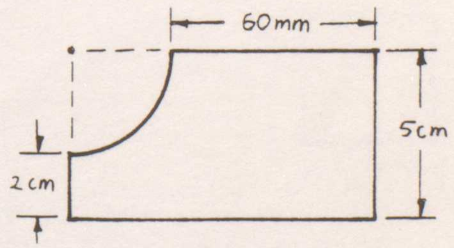
h)



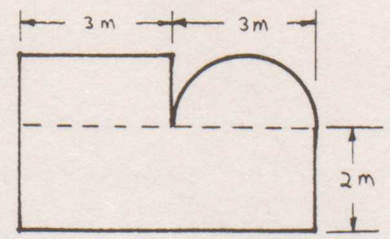
i)



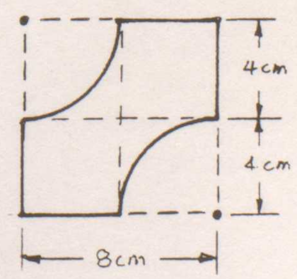
j)



k)



l)



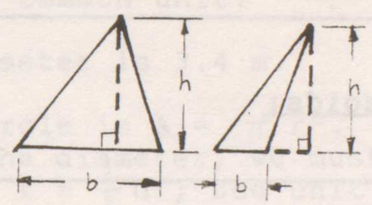
UNIT 9

UNIT 9: AREA PROBLEMS

We already discussed what area is in a previous unit. We will now concentrate on solving area problems. As for perimeter problems, we must remember to have all measurements in the same units when we find the area of figures. For some basic shapes, we have special formulas that will enable us to find their area. No proofs are given for the formulas, but some of them are obvious. The formulas that are not so obvious will be explained in your present or future math classes.

Triangles:

For triangles:

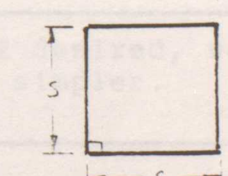


$$A = \frac{1}{2} \times b \times h$$

Where A = area of the triangle
b = base of the triangle
h = height of the triangle

Squares:

For squares:

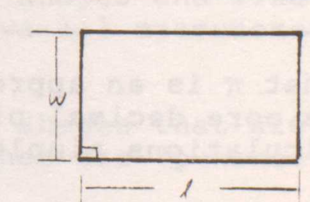


$$A = s^2$$

Where A = area of the square
s = length of the side (s^2 means s x s)

Rectangles:

For rectangles:

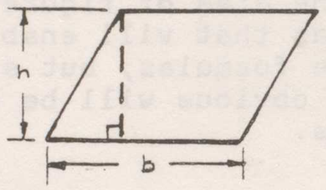


$$A = l \times w$$

Where A = area of the rectangle
l = length of the rectangle
w = width of the rectangle

Parallelograms:

For parallelograms:

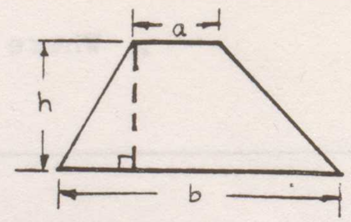


$$A = b \times h$$

Where A = area of the parallelogram
b = base of the parallelogram
h = height of the parallelogram

Trapezoids:

For trapezoids:

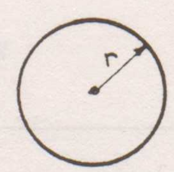


$$A = \frac{1}{2} h(a + b)$$

Where A = area of the trapezoid
h = height of the trapezoid
a , b = the lengths of the parallel sides

Circles:

For circles:



$$A = \pi r^2$$

Where A = area of the circle
π = constant (approximately = 3.14)
r = radius of the circle (r^2 means r x r)

Note that π is an approximation. If we desire greater accuracy, we can use more decimal places for π (3.141592654...). However, to keep our calculations simple, we will use 3.14.

EXAMPLE 1: Find the area of a rectangle with a length of 56 mm and a width of 3 cm.

We recall that the area of a rectangle is equal to its length times its width. However, we must have all measurements in the same units, so we must change 56 mm into 5.6 cm. Now we can apply the formula:

$$A = l \times w$$

$$A = 5.6 \text{ cm} \times 3 \text{ cm}$$

$$A = 16.8 \text{ cm}^2$$

Note that we could have changed all measurements into mm, but we prefer to have things in cm^2 since it is a more common unit.

EXAMPLE 2: Find the area of a circle whose diameter is 3.4 m.

We recall that the formula for the area of a circle is $A = \pi r^2$. Since the formula requires the radius and not the diameter, we must find the radius. After doing our calculation ($r = \frac{1}{2} d$; see unit on perimeter), we see that the radius is 1.7 m. We can now apply our formula:

$$A = \pi \times r^2$$

$$A = 3.14 \times (1.7 \text{ m})^2$$

$$A = 3.14 \times 2.89 \text{ m}^2 \quad (\text{note: } 1.7^2 = 1.7 \times 1.7)$$

$$A = 9.0746 \text{ m}^2$$

Therefore, the area of the circle is 9.0746 m^2. If desired, we can round the answer off to 9.1 m^2 to make things look simpler.

COMPLEX SHAPES:

Very often, we are required to find the area of figures that are not one of the simple shapes with a formula we can readily use. To solve these particular cases, we can follow the guidelines below:

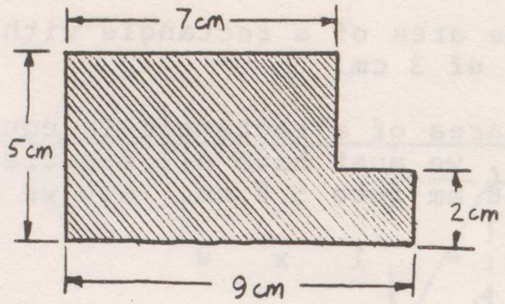
1) Visualize how the shape can be obtained, either by combining one or more of the basic shapes and/or by "punching out" one or more of the basic shapes.

2) Find the dimensions for each of the basic shapes and find the area of each basic shape. Remember to have all measurements in the same units.

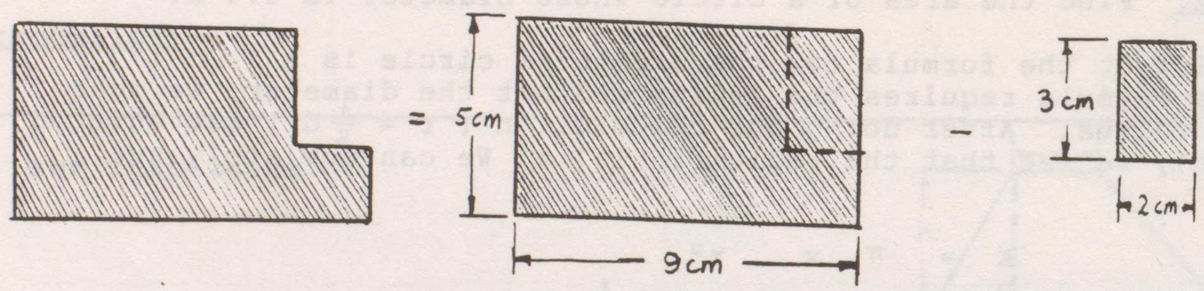
3) Find the resulting area by adding all the shapes that are combined and subtracting all the shapes that are "punched out".

Note that there are usually several possible ways of finding the area of a complex figure.

EXAMPLE 3: Find the area of the following region:



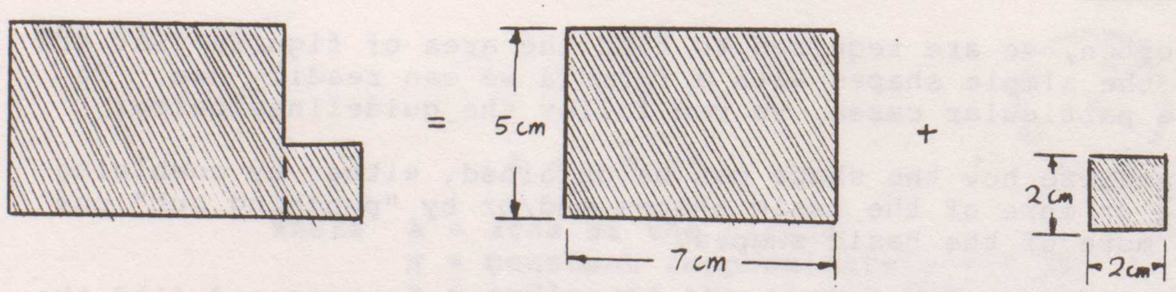
We see that the above figure can be obtained by taking the entire rectangle and "punching out" the top right corner. Below is a diagram of how this is done as well as the dimensions of each of the basic shapes (the basic shapes are rectangles in this case and, hence, we can find the area of each rectangle by multiplying the length with the width):



shaded area = 45 cm² − 6 cm²

= 39 cm²

We could have also approached this problem in a slightly different manner. We see that the figure is actually a combination of two rectangular shapes. Below is a diagram illustrating this as well as the dimensions of the two rectangular components:

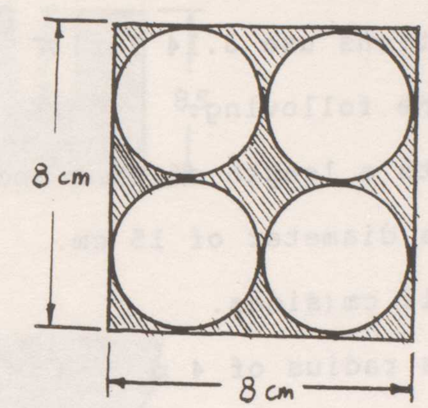


shaded area = 35 cm² + 4 cm²

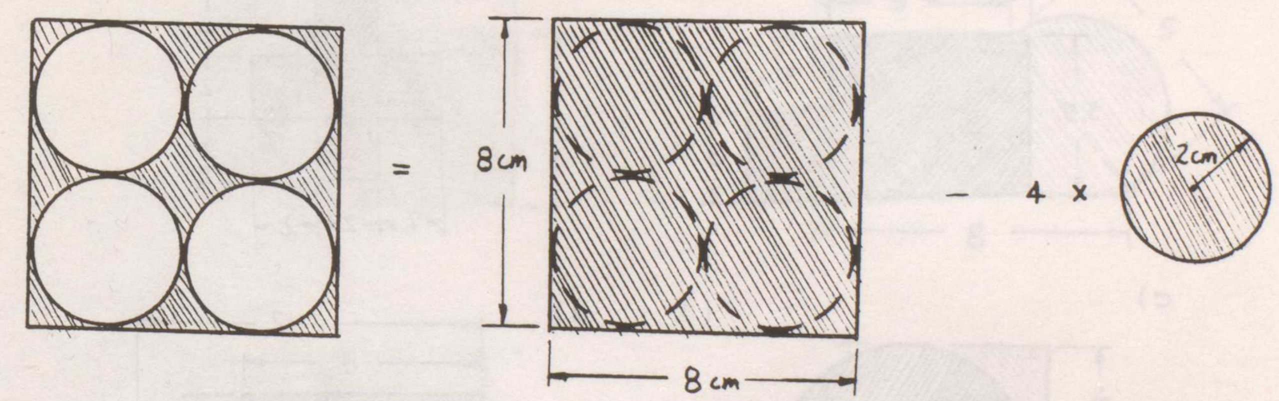
= 39 cm²

Thus, the area of the shaded region is equal to 39 cm².

EXAMPLE 4: Find the area of the shaded region:



We see that this particular area can be found by taking a square and then "punching out" the four circular shapes. We notice that in this case the four circles are the same size (we also see that each circle has a diameter of 4 cm and, thereby, we can calculate and find that the radius is equal to 2 cm). Below is a diagram illustrating the process of finding the shaded region:



Shaded area = (s^2) − 4 x (π r^2)

= 64 cm² − 4 x (12.56 cm²)

= 13.76 cm²

Therefore, the area of the shaded region is equal to 13.76 cm²

EXERCISE 9: AREA PROBLEMS

Note: for all the questions use 3.14 for π (if necessary).

1) Find the areas of the following:

   a) a rectangle with a length of 19 m and a width of 5 m.

   b) a circle with a diameter of 15 cm.

   c) a square with 12 cm sides.

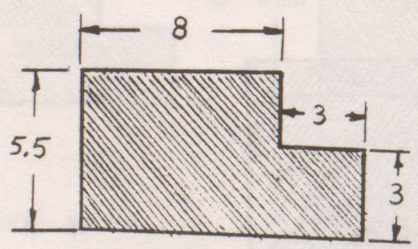
   d) a circle with a radius of 4 m.

   e) a triangle whose base is 24 cm and height is 12 cm.

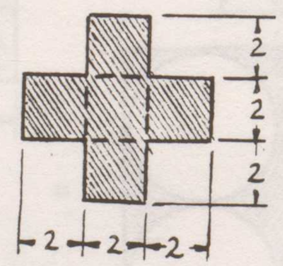
   f) Find the area of a rectangle with a length of 100 m and width of 900 cm.

2) Find the area of the shaded regions (all measures are in cm):

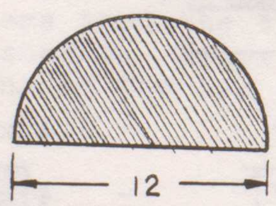
a)



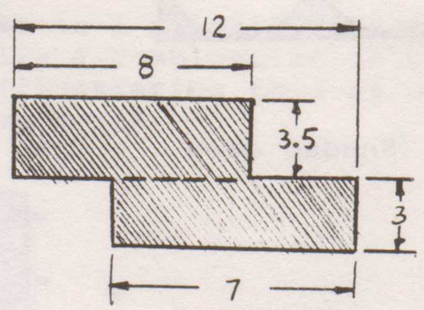
b)



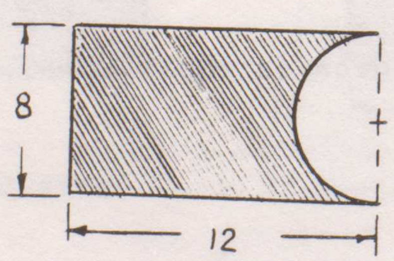
c)



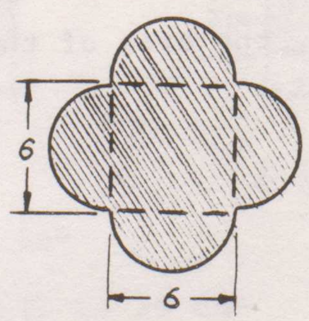
d)



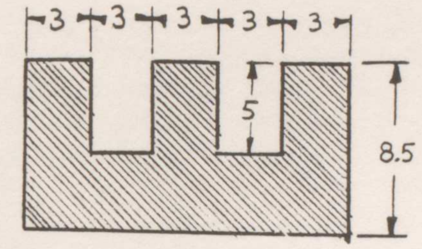
e)



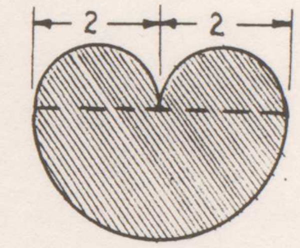
f)



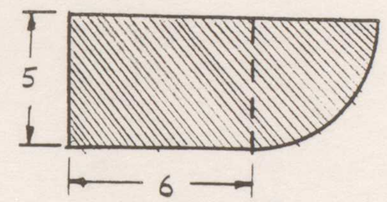
g)



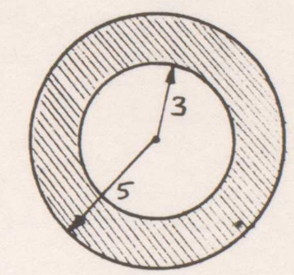
h)



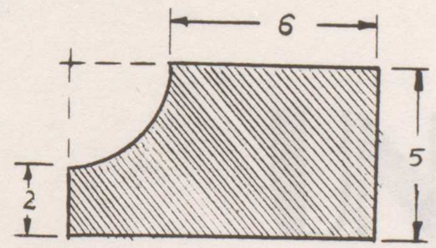
i)



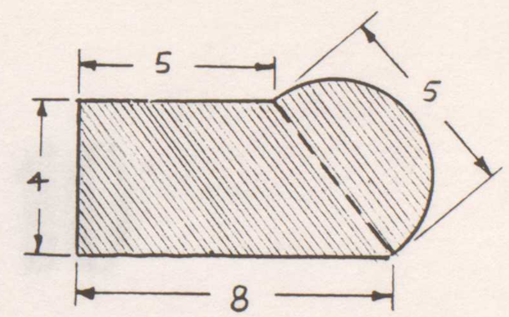
j)



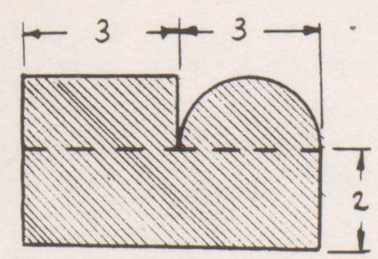
k)



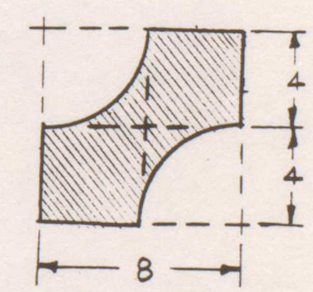
l)



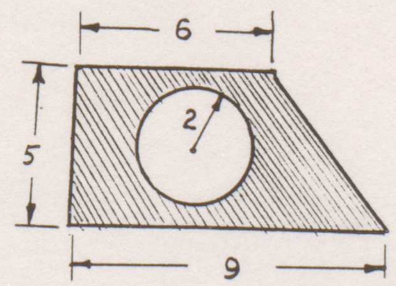
m)



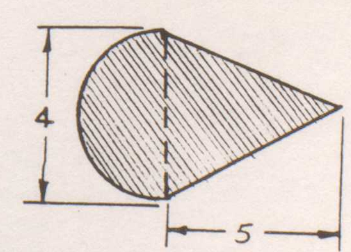
n)



o)



p)



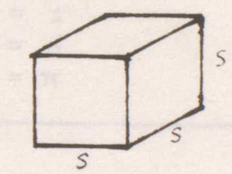
UNIT 10

UNIT 10: VOLUME PROBLEMS:

As for perimeter and area problems, there are special formulas for calculating the volume of some basic shapes. No proofs will be given for the formulas; some are obvious and those that are not will be explained in your present or future math classes. Below are the formulas for some basic shapes:

Cubes:

For Cubes:

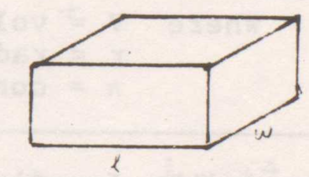


$$V = s^3$$

Where V = Volume of the cube
s = length of the sides (s^3 means $s \times s \times s$)

Rectangular Solids:

For rectangular solids:

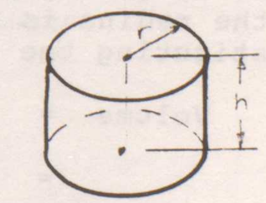


$$V = l \times w \times h$$

Where V = volume of the rectangular solid
l = length of the rectangular solid
w = width of the rectangular solid
h = height of the rectangular solid

Right Circular Cylinders:

For right circular cylinders:

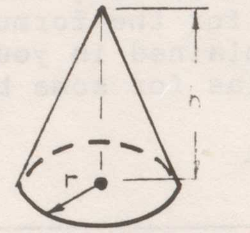


$$V = \pi r^2 h$$

Where V = volume of the cylinder
r = radius of the circular end (r^2 means $r \times r$)
h = height of the cylinder
π = constant (approximately = 3.14)

Right Circular Cones:

For right circular cones:

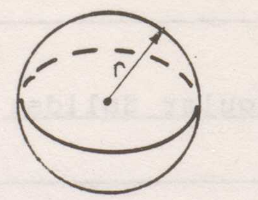


$$V = \frac{1}{3} \pi r^2 h$$

Where V = volume of the cone
r = radius of the circular end (r^2 means r x r)
h = height of the cone
π = constant (approximately = 3.14)

Spheres:

For spheres:



$$V = \frac{4}{3} \pi r^3$$

Where V = volume of the sphere
r = radius of the sphere (r^3 means r x r x r)
π = constant (approximately = 3.14)

Note that in the formulas, we can obtain greater accuracy by using more decimal places for π (3.141592654...) if it is part of the formula. "Right" means the shape is not at a slant. For the cube, rectangular solid, and right circular cylinder, we can remember the formula by thinking of volume as being equal to the area of the base times the height (i.e. V = A x h).

EXAMPLE 1: Find the volume of a sphere with a diameter of 12 cm.

We see that the formula for the sphere involves the radius. Since the diameter is given, we can easily find the radius as being equal to 6 cm (the radius is equal to half the diameter). We can now proceed by substituting the value of the radius in our formula:

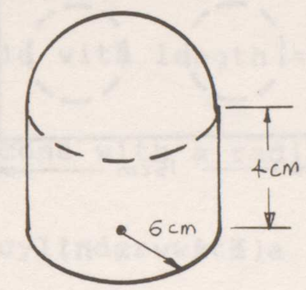
$$\text{Volume} = \frac{4}{3} \times \pi \times r^3$$
$$= \frac{4}{3} \times 3.14 \times (6\text{ cm})^3$$
$$= \frac{4}{3} \times 3.14 \times (216\text{ cm}^3) \quad \text{Note: } 6^3 = 6 \times 6 \times 6$$
$$= 904.32\text{ cm}^3$$

Therefore, the volume of the sphere is 904.32 cm^3.

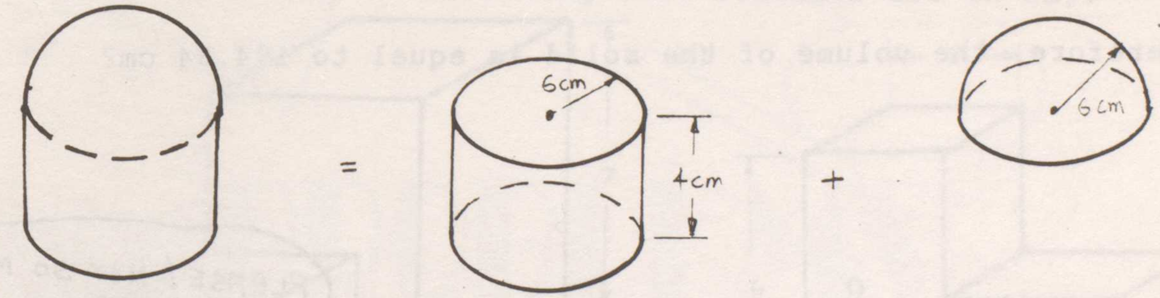
Complex Shapes:

To find the volume of complex shapes, we must break our particular solid down as a combination of basic shapes and/or a "punching out" of basic shapes.

EXAMPLE 2: Find the volume of the following solid:



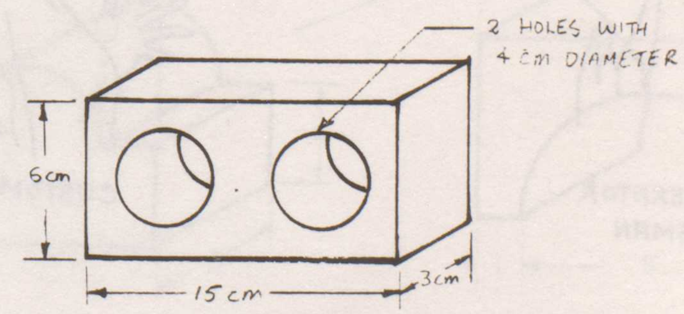
We see that the solid is made up of a right circular cylinder and half of a sphere. We can visualize and calculate the volume as below:



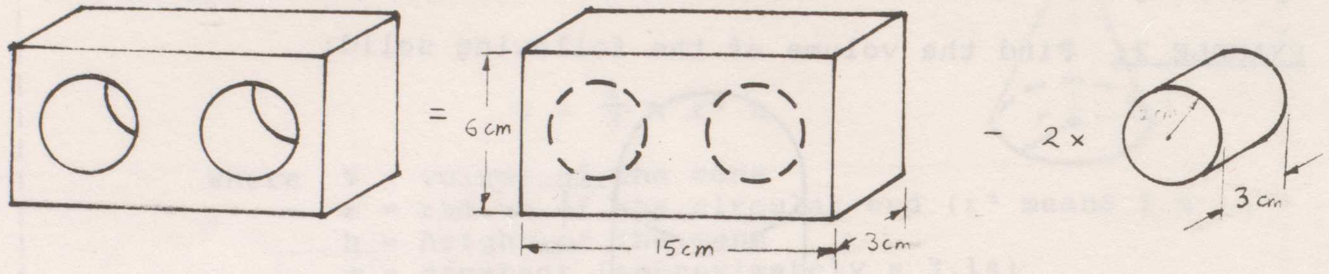
$$\text{Volume of solid} = (\pi\ r^2\ h) + \frac{1}{2} \times \left(\frac{4}{3}\ \pi\ r^3\right)$$
$$= 452.16\text{ cm}^3 + 452.16\text{ cm}^3$$
$$= 904.32\text{ cm}^3$$

Therefore, the volume of the object is 904.32 cm^3.

EXAMPLE 3: Find the volume of the following figure:



We see that the volume of the solid can be found by taking the volume of the rectangular solid and then "punching out" the volume of two right circular cylinders. We can visualize this and calculate the volume as below:

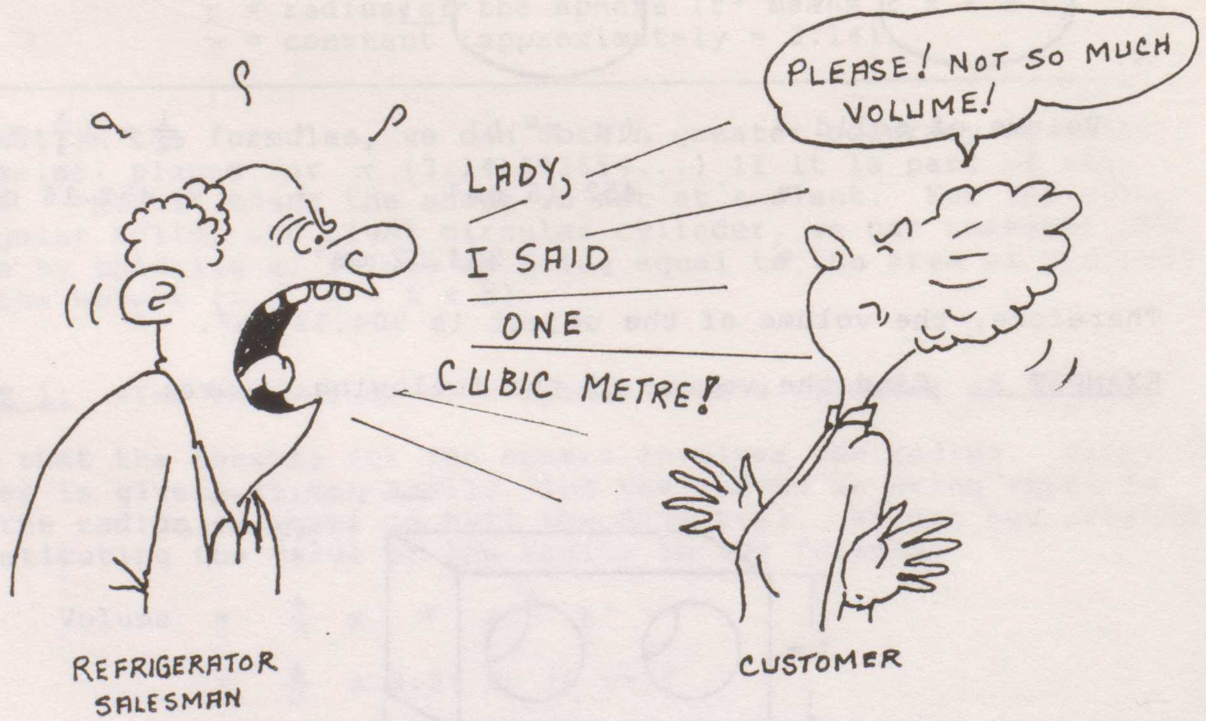


Volume of solid = $(l \times w \times h) \quad - \quad 2 \times (\pi r^2 h)$

$= 270 \text{ cm}^3 \quad - \quad 2\ (37.68 \text{ cm}^3)$

$= 194.64 \text{ cm}^3$

Therefore, the volume of the solid is equal to 194.64 cm³



EXERCISE 10: VOLUME PROBLEMS

Note: use 3.14 for π.

1) Find the volume of the given solid:

   a) a cube with 5 cm sides.

   b) a rectangular solid with length = 3.5 cm, width = 2 cm, and height = 4.5 cm.

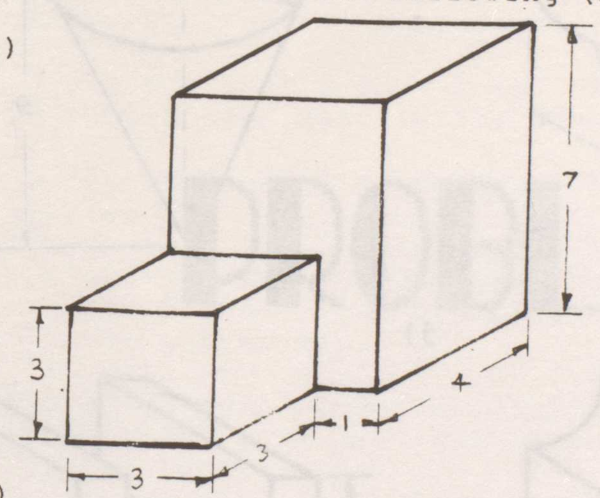
   c) a right circular cone with a radius of 6 cm and a height of 13.25 cm.

   d) a right circular cylinder with a diameter of 7 cm and a height of 3 cm.

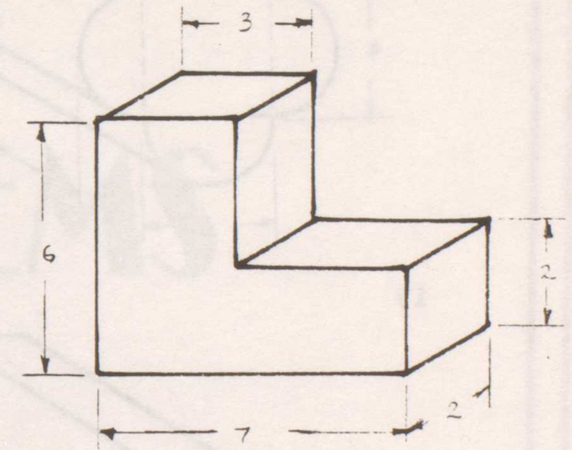
   e) a sphere with a radius of 9 cm.

2) Find the volume of the following (all measures are in cm):

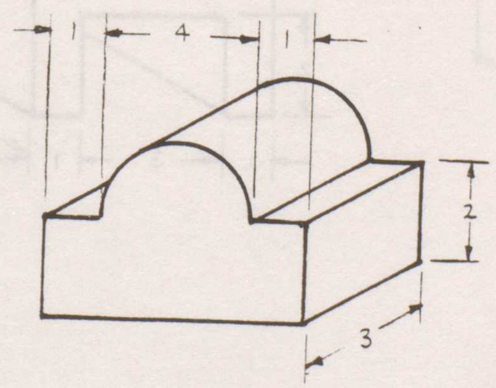
a)



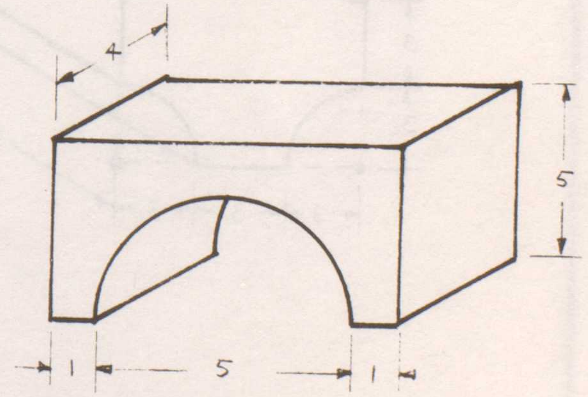
b)



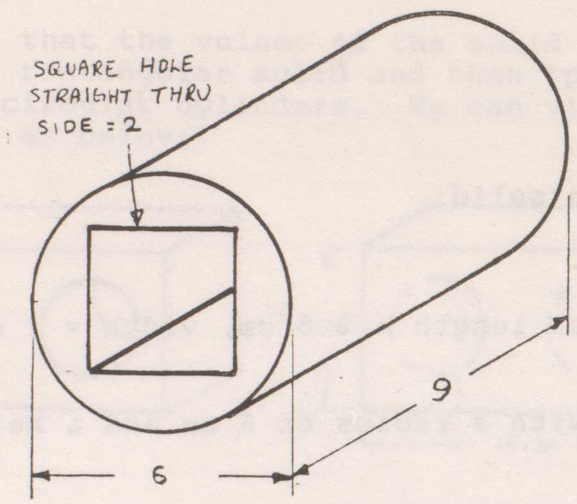
c)



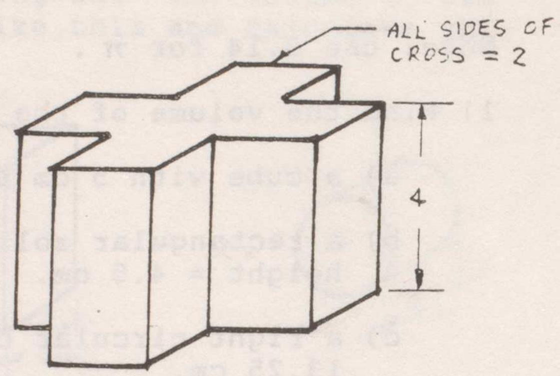
d)



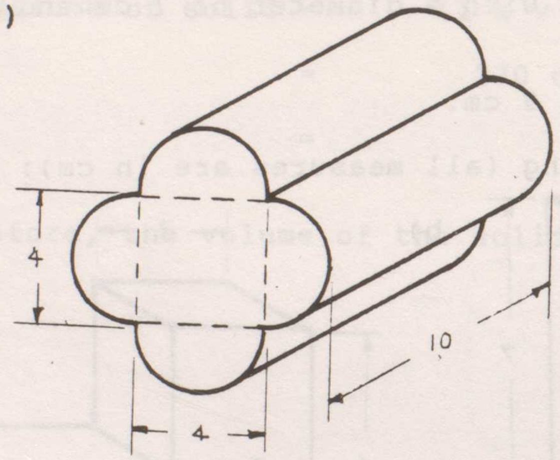
e)



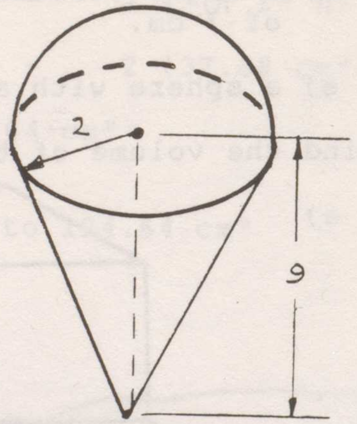
f)



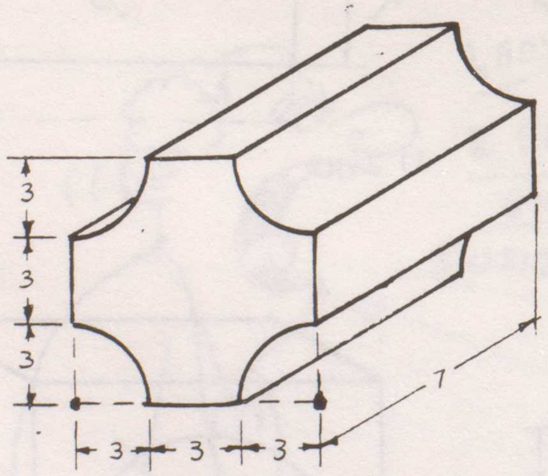
g)



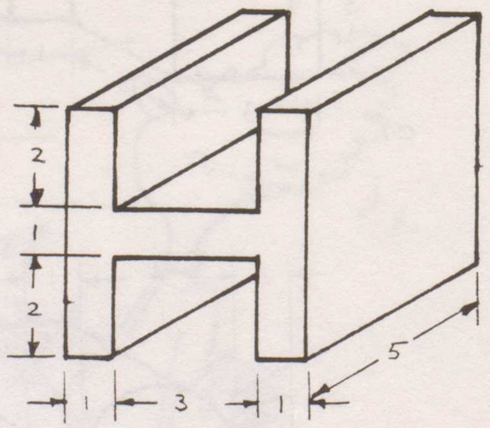
h)



i)



j)



PROBLEMS

PROBLEMS

1) How many 15 ml doses can be obtained from a 255 ml bottle of medicine?

2) There is 4 mg of Bromphenramine maleate per 5 ml of a cold medicine. How many mg are there in a 150 ml bottle?

3) A certain car gets 100 km / 8.9 L of gasoline. How far can the car travel on a 45 L tank of gasoline?

4) It takes Billy 8.5 seconds to run travel 50 m. How long would it take him to travel 250 m?

5) Gasoline costs 49.9 cents per litre. How much would it cost for a 42.35 L fill-up?

6) Mushrooms cost $2.59 / kg. How many kg can you purchase for $10.00?

7) Silk costs $25.99 / m. How much would it cost to make a silk dress that requires 2.4 m of fabric?

8) A map has a scale of 3 cm = 1 km. If you measure a distance of 15.5 cm on the map, what is the actual distance?

9) A 500 g bag of cookies contains 45 cookies. What is the mass of each cookie?

10) During a sale, a 750 ml bottle of pop costs 69 cents and a 2 L bottle costs $1.68. Find the cost in cents / ml and determine which is the better buy.

11) It costs $6.48 for 2 L box of detergent A and $4.88 for 1.4 L box of detergent B. Find the cost in dollars per litre and determine which is the better buy.

12) Cooked shrimp costs $1.32 / 100 g. How much would 1.95 kg cost?

13) Sea water contains 1.7 g of salt per 500 ml of water. How many grams of salt are in 4.5 L of sea water?

14) Mouthwash A costs 79 cents for 175 ml and mouthwash B costs $4.69 for 750 ml. Find the cost in cents per ml and determine which is the better buy.

15) A 500 ml bottle of bubble bath costs $1.29. If each bath requires about 24 ml of bubble bath, about how many baths will the bottle last?

16) The sale price of a 500 ml bottle of dishwashing liquid is 97 cents and the regular price of a 1 L bottle is $2.19. If you bought 4 L of liquid, how much money will you save by buying the 500 ml bottle size?

17) Soap A comes in packages of three 90 g bars and costs $1.44. Soap B comes in packages of two 80 g bars and costs 99 cents. Find the cost in cents / g and determine which is the better buy.

18) An elevator can support a load of 1.5 tonnes. How many people can it support assuming that each person has a mass of 70 kg?

19) One metre of fencing costs $9.00. How much would it cost to fence a rectangular property that has a width of 10 m and a length of 35 m?

20) A 10.5 m by 37.5 m rectangular plot of land is purchased for $89 000. What is the cost per square metre?

21) The cooking instructions suggests 5 L of water and 15 ml of salt to cook 500 g of pasta. How much water and how much salt should be used to cook 800 g of pasta?

22) Every 500 g of dry spaghetti yields 8 L of cooked product. How many servings is this if each serving is 325 ml?

23) Using the information in question 22, how many grams of dry spaghetti should be used to make 45 servings?

24) A particular brand of tile costs 84 cents each and measures 30 cm by 30 cm. How much would it cost to tile a floor that is 9 m by 9 m. How much would it cost with a sales tax of 6%?

25) A 2 L bottle of mineral water costs $1.89. Winky's hot tub is a right circular cylinder with a radius of 1 m and a height of 1 m. How much would it cost to fill half the tub with mineral water?

APPENDIX

TEMPERATURE:

In 1742, Anders Celsius, a Swedish astronomer, devised a temperature scale by selecting 0 as the freezing point of water and 100 as the boiling point of water. He called this the centigrade (100 grades) thermometer. In recent years, however, the name has been changed to Celsius in honor of the inventor. Out of interest, the Farenheit system was devised by a German instrument maker named Gabriel Farenheit in 1714; he was the inventor of the first mercury thermometer. Gabriel called the lowest temperature he could attain with ice and salt 0 degrees, and the normal human body temperature of 96 degrees for the upper point of his scale (this should have actually been 98.6). On this scale, water freezes at 32 degrees and boils at 212 degrees. The metric system uses the Celsius scale for temperature. For the few of us who are still not familiar with the Celsius scale, here are some temperatures:

Water freezes	:	0° C
Room temperature	:	21° C
Warm day	:	25° C
Body temperature	:	37° C
Bath water	:	45° C
Water boils	:	100° C

We can convert between the two systems by using the following formulas:

$$C = 5/9\ (F - 32)$$

$$F = 9/5\ C + 32$$

Suppose the temperature in Vancouver is 12° C. To convert to Farenheit, we use the Farenheit formula, $F = 9/5\ C + 32$, replacing C with 12. This gives the following:

$$F = 9/5\ (12) + 32 = 53.6^\circ\ F \text{ (or about } 54^\circ\ F)$$

Suppose the temperature in Los Angeles is 68° F. To convert, we use the Celsius formula, $C = 5/9\ (F - 32)$, replacing F with 68. This gives the following:

$$C = 5/9\ (68 - 32) = 20^\circ\ C$$

Remember to follow the Order of Operations when you do your calculations.

EXERCISES:

a) 10° C = ________ F	f) 10° F = ________ C
b) -20° C = ________ F	g) -20° F = ________ C
c) 36° C = ________ F	h) 36° F = ________ C
d) 84° C = ________ F	i) 84° F = ________ C
e) -40° C = ________ F	j) -40° F = ________ C

OTHER METRIC PREFIXES:

The prefixes mentioned in the units are not the only prefixes in the metric system. Sometimes (especially in Science), we need to express units that are much larger or smaller than the base unit. The following is a summary of the prefixes and the commonly used ones are indicated by an asterisk (*).

Prefix	Symbol	Factor	
exa	E	1 000 000 000 000 000 000	(10^{18})
peta	P	1 000 000 000 000 000	(10^{15})
tera	T	1 000 000 000 000	(10^{12})
giga	G	1 000 000 000	(10^{9})
* mega	M	1 000 000	(10^{6})
* kilo	k	1 000	(10^{3})
hecto	h	100	(10^{2})
deka	da	10	
deci	d	0.1	(10^{-1})
* centi	c	0.01	(10^{-2})
* milli	m	0.001	(10^{-3})
* micro		0.000001	(10^{-6})
* nano	n	0.000000001	(10^{-9})
* pico	p	0.000000000001	(10^{-12})
femto	f	0.000000000000001	(10^{-15})
atto	a	0.000000000000000001	(10^{-18})

CONVERSION USING THE CALCULATOR:

For those of us who have difficulty with the unit fraction method of conversion or those of us who are just lazy, the following may be useful in helping us convert with our calculators. Simply enter the numbers and symbols in the order presented below. The first measure indicates the starting units (enter the appropriate number) and the second measure is the units of the calculator display. Note that these are approximations only.

IMPERIAL TO METRIC:

Length:

inches	x	2.54	=	centimetres
feet	x	30.48	=	centimetres
yards	x	0.9144	=	metres
miles	x	1.60934	=	kilometres

Capacity:

pints	x	0.56826	=	litres
quarts	x	1.13652	=	litres
gallons	x	4.54609	=	litres
(U.S.) pints	x	0.47318	=	litres
(U.S.) quarts	x	0.94635	=	litres
(U.S.) gallons	x	3.78541	=	litres

Mass:

ounces	x	28.3495	=	grams
pounds	x	0.45459	=	kilograms

METRIC TO IMPERIAL:

Length:

centimetres	x	0.3937	=	inches
centimetres	x	0.03281	=	feet
metres	x	1.09361	=	yards
kilometres	x	0.62137	=	miles

Capacity:

litres	x	1.75976	=	pints
litres	x	0.87988	=	quarts
litres	x	0.21997	=	gallons
litres	x	2.11338	=	pints (U.S.)
litres	x	1.05669	=	quarts (U.S.)
litres	x	0,26417	=	gallons (U.S.)

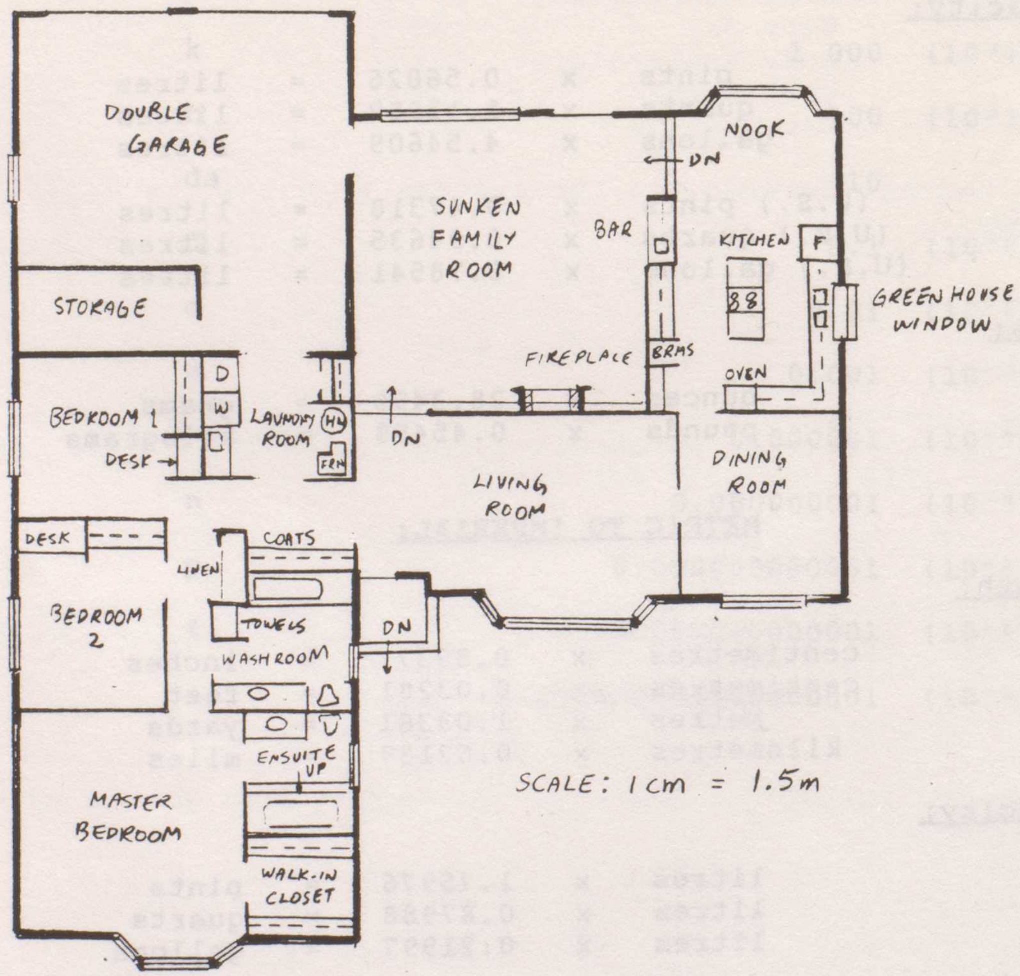
Mass:

grams	x	0.03527	=	ounces
kilograms	x	2.20462	=	pounds

PRACTICAL APPLICATION

Most problems in real life may not be solved with 100% accuracy. However, we should be able to obtain reasonable approximations using mathematical formulas and calculations. Situations in real life may not always work out as nicely as in most text book problems. The following questions are based on a real life situation; see if you can use your mathematical knowledge to come up with sensible estimates. Refer to the scale drawing of the rancher and the list of prices to answer the questions.

SCALE DRAWING OF RANCHER:



SCALE: 1cm = 1.5m

LIST OF PRICES:

CARPET: $69.97 / running metre

VINYL FLOORING: $47.21 / running metre

Note: Running metre means that you must buy entire sheets cut from a roll that is 3.66 m wide. Cuts are made to the nearest 0.1 of a metre.

VINYL TILES: $116.55 / package of 45 (each tile is 30 cm by 30 cm)

WALL PAPER: $14.98 / package of 2 bolts (each bolt measures 4.5 m by 9 m)

Note: You must buy complete packages.

EXTERIOR PAINT: $45.99 / 4 L can

Note: You must buy complete cans. Each can of paint covers an area of about 40 square metres.

QUESTIONS:

1) Find the approximate dimensions of the following rooms to the nearest 0.1 of a metre: Double Garage (and Storage), Bedroom 1, Bedroom 2, Master Bedroom (including closet and ensuite), Washroom, Laundry Room, Living Room, Dining Room, Family Room and Kitchen (including Nook).

2) Find the total area of the house (including garage).

3) Find the total cost to put:

   a) carpet in the Master Bedroom and Walk-In Closet, Bedroom 1, Bedroom 2, Living Room, Dining Room, Family Room, and Hallways.

   b) vinyl flooring in the Kitchen and Nook, and Laundry Room.

   c) vinyl tiles in the Washroom and Ensuite.

   d) wallpaper in Bedroom 1 and Bedroom 2 (assume walls are 2.6 m high).

   e) paint on the exterior walls of the house (assume all the exterior walls are 3 m high).

4) Find the total cost of all the renovations done in question 3 and account for a sales tax of 6% on all the purchases.

ANSWERS

<u>EXERCISE 1A:</u> (Pages 13 & 14)

1)	a) 1000 of b) 0.001 of	c) 100 of d) 0.01 of	e) 10 of f) 0.1 of
2)	a) mm b) dam c) m	d) km e) hm	f) dm g) cm
3)	a) capitals; km b) period; m	c) period; cm d) capital; hm	e) missing base; km
4)	a) mm b) km c) m	d) m e) cm	f) cm g) mm
5)	a) cm b) km c) mm	d) m e) km	f) m g) cm

<u>EXERCISE 1B:</u> (Page 16)

1)	a) ml b) L c) dal	d) dl e) hl	f) kl g) cl
2)	a) ml b) L	c) ml	d) kl
3)	a) L b) ml	c) L d) ml	e) L

<u>EXERCISE 1C:</u> (Page 18)

1)	a) kg b) g c) hg	d) dg e) cg f) dag	g) mg h) t
2)	a) t b) g	c) mg d) kg	e) kg
3)	a) mg b) g	c) kg d) t	e) g f) kg

EXERCISE 2A: (pages 27 & 28)

1)

a) 2300	h) 0.1	o) 1000
b) 478	i) 0.1	p) 10
c) 0.097	j) 0.4	q) 0.01
d) 7.5	k) 6.76	r) 0.001
e) 0.06	l) 6.775	s) 60
f) 100	m) 250	t) 68.8
g) 100 000	n) 9.487	

2)

a) 1000	h) 576 900	o) 0.642
b) 0.001	i) 320	p) 5680
c) 1000	j) 0.00087	q) 0.042
d) 0.001	k) 0.5689	r) 3.212
e) 0.000001	l) 4780	s) 4000
f) 1 000 000	m) 0.03075	t) 0.00075
g) 4.2	n) 570 000	

3)

a) 1000	h) 89	o) 0.004569
b) 1000	i) 40 000	p) 5
c) 1000	j) 85 000	q) 400 000
d) 0.001	k) 0.2	r) 0.6505
e) 0.001	l) 0.045	s) 10 000
f) 0.001	m) 103 800	t) 0.084
g) 0.00745	n) 9200	

4)

a) 5600	h) 29 000	o) 0.000002
b) 750	i) 350	p) 35 000
c) 4560	j) 0.013	q) 0.1
d) 750	k) 0.07	r) 0.3454
e) 5.025	l) 0.00035	s) 2 255 500
f) 2500	m) 0.0001956	t) 13
g) 10 000	n) 8 000 000	

EXERCISE 2B: (Page 29)

1)

a) 78	j) 4 560 000	s) 0.003578
b) 56 600	k) 3.4	t) 6.7635
c) 20	l) 0.4	u) 8
d) 9800	m) 0.03	v) 90 000
e) 845 100	n) 2300	w) 675 500 000
f) 45	o) 62.78	x) 5647.5
g) 637.5	p) 6.776	y) 3.5987
h) 30	q) 0.004595	z) 420
i) 9999	r) 233	

EXERCISE 3: (Page 36)

1)

a) m^2	c) ha	e) m^2
b) cm^2	d) km^2	

2)

a) 20 800	f) 6	k) 17.4
b) 150	g) 47 500 000	l) 5.6
c) 4	h) 20 000 000	m) 18 000
d) 7.5	i) 0.056	n) 900 000
e) 0.025	j) 427	

3)

a) 10 000	h) 20	o) 765
b) 38	i) 786 000	p) 6.5
c) 6000	j) 4 750 000	q) 60 000
d) 201 000	k) 0.0001	r) 9.8
e) 20 000 000	l) 47 000	s) 2.5
f) 2000	m) 60	t) 475
g) 27 000	n) 0.1	

EXERCISE 4 (Page 41)

1)

a) m^3	c) cm^3	d) m^3
b) cm^3		

2)

a) 5 700 000	j) 700	s) 678.9
b) 0.000345	k) 6 400 000	t) 0.002
c) 4500	l) 0.02	u) 1 000 000
d) 270	m) 27 600	v) 196 000
e) 0.005673	n) 5	w) 500 000
f) 227 000 000	o) 575 000	x) 5 740 000 000
g) 0.5	p) 567.5	y) 0.3
h) 1000	q) 4 500 000	z) 0.005
i) 0.00092	r) 0.052768	

EXERCISE 5: (Page 46)

1)

a) 37	e) 8.76	i) 4.567
b) 455	f) 0.05325	j) 0.000235
c) 32 578	g) 45 700	k) 5 690 000
d) 4.76	h) 750 000	l) 495 000

2)

a) 3 kg	d) 80 m^3	g) 10 L
b) 200 000 m^3	e) 80 000 kg	h) 8 L
c) 200 000 t	f) 250 g	

EXERCISE 6: (Page 50) (Note: some answers are rounded off.)

1)	a) 4840	g) 7.5	m) 20.94
	b) 18.8	h) 2.5	n) 4600
	c) 64.5	i) 28	o) 4.42
	d) 0.375	j) 24	p) 11.25
	e) 1620	k) 3.75	
	f) 63 360	l) 7204	

2)	Metric		
	a) 0.24	f) 125	k) 0.92
	b) 2800	g) 21.12	l) 1.44
	c) 0.85	h) 6000	m) 1.2
	d) 1200	i) 0.33	
	e) 18 000	j) 23.760	
	Imperial		
	a) 2	f) 3.75	k) 7.7
	b) 336	g) 12	l) 12
	c) 255	h) 10 560	m) 432
	d) 4	i) 2.75	
	e) 31 680	j) 4.5	

EXERCISE 7: (Page 57) (Note some answers may be rounded off.)

1)	a) 8.89	e) 28.96	i) 20.67
	b) 78.74	f) 2.33	j) 32.18
	c) 9.14	g) 114.84	
	d) 2.21	h) 5.21	
2)	a) 0.0045	e) 500 000	i) 560
	b) 275	f) 350 000	j) 0.0578
	c) 130	g) 0.00405	
	d) 167 000	h) 0.004567	
3)	a) 1287.2 km	c) 8839.8 m	e) 5 ft.
	b) 8.9 m	d) 7 ft. 4 in.	f) 88.5 km/h

EXERCISE 8: (Pages 68 & 69) (Note some answers may be rounded off.)

1)	a) 30 cm	d) 44 cm	g) 31.4 m
	b) 10.2 m	e) 52 cm	h) 36 cm
	c) 94.2 mm	f) 154 cm	i) 11.1 cm
2)	a) 9 cm	c) 15 sides	d) 5.7 m
	b) 10 m		
3)	a) 33 cm	e) 67 m	i) 12.56 cm
	b) 44.56 m	f) 30.84 cm	j) 26.71 cm
	c) 37.68 cm	g) 37 cm	k) 20.71 m
	d) 24.85 m	h) 29.85 m	l) 28.56 cm

EXERCISE 9: (Pages 78 & 79) (Note some answers may be rounded off.)

1)	a) 95 m^2	c) 144 cm^2	e) 144 cm^2
	b) 176.6 cm^2	d) 50.24 m^2	f) 900 m^2

2) (Note all answers are in cm^2.)

a) 53	g) 97.5	m) 20.03
b) 20	h) 9.42	n) 38.88
c) 56.52	i) 49.63	o) 24.94
d) 49	j) 50.24	p) 16.28
e) 70.88	k) 37.94	
f) 92.52	l) 35.81	

EXERCISE 10: (Pages 87 & 88) (Note some answers may be rounded off.)

1)	a) 125 cm^3	c) 499.26 cm^3	e) 3052.08 cm^3
	b) 31.5 cm^3	d) 115.40 cm^3	

2) (Note all answers are in cm^3)

a) 139	e) 218.34	i) 369.18
b) 52	f) 80	j) 65
c) 54.84	g) 411.2	
d) 100.75	h) 54.43	

PROBLEMS: (Pages 91 & 92) (Note some answers are rounded off.)

1) 17 doses

2) 120 mg

3) 506 km

4) 42.5 s

5) $21.13

6) 3.9 kg

7) $62.38

8) 5.17 km

9) 11.1 g

10) 750 ml - 0.092¢/ml
2 L - 0.084¢/ml
2 L is better buy

11) A - $3.24/L
B - $3.49/L
A is better buy

12) $25.74

13) 15.3 g

14) A - 0.45¢/ml
B - 0.63¢/ml
A is better buy

15) 21

16) $1.00

17) A - 0.53¢/g
B - 0.62¢/g
A is better buy

18) 21

19) $810

20) $226.03 / m^2

21) water - 8 L
salt - 24 ml

22) 24.6

23) 914 g

24) $756, $801.36

25) $1483.65

TEMPERATURE (APPENDIX): (Page 95)

a) 50°
b) -4°
c) 96.8°
d) 183.2°
e) -40°
f) -12°
g) -29°
h) 2°
i) 29°
j) -40°

PRACTICAL APPLICATION: (Pages 98 & 99) (The answers are approximations only! Your answers may be different.)

1) All measurements are in m: Garage - 6.5 by 6.6 , Bedroom 1 - 3.2 by 3.6 , Bedroom 2 - 2.9 by 3.8 , Master Bedroom - 6.6 by 4.4 , Washroom - 2.7 by 2.7 , Laundry -2.7 by 2.4 , Living - 6.3 by 3.6 , Dining 3.2 by 3.6 , Family - 6.3 by 5.4 , Kitchen 3.8 by 5.7 .

2) 215 square metres.

3) a) 34.5 running metres; $2414
b) 7.9 running metres; $373
c) 2 packages; $233
d) 5 packages; $75
e) 6 cans; $276

4) $3573